

M-Theory on Twisted Connected Sum G_2 -Manifolds

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Introduction

- M-Theory on 7d manifolds with G_2 -holonomy
 - ▶ 4d low energy effective theory with minimal SUSY
 - ▶ Semi-classical unique 11d SUGRA action
- Technical challenges of G_2 -compactifications
 - ▶ Compact 7d G_2 -manifolds are difficult to describe explicitly
 - ▶ Up to fairly recently: Not many compact G_2 -manifolds known
- Kovalev's twisted connected sum G_2 -manifolds
 - ▶ Compact G_2 -manifolds from CY 3-fold building blocks
 - ▶ Kovalev limit of G_2 -manifolds:
Gauge Theories via algebraic geometry
 - ▶ Proposal from Gauge Theory:
Controlled Geometric transitions among G_2 -manifolds

References

○ Physics:

► M-Theory & G_2 -manifolds:

Acharya ('96), Harvey-Moore ('99), Acharya-Spence ('00), Atiyah-Maldacena-Vafa ('00), Atiyah-Witten ('01), Witten ('01), Acharya-Witten ('01), Aganagic-Vafa ('01), Beasley-Witten ('02), Berglund-Brandhuber('02), Gukov-Yau-Zaslow ('02), Lukas-Morris('03), House-Micu ('04), Anderson-Barret-Lukas-Yamaguchi ('06), Font ('10), ...

► Kovalev's Twisted Connected Sum G_2 -manifolds:

Halverson-Morrison ('15)&('15), Braun ('16), Braun-Del Zotto ('17)

○ Mathematics:

► G_2 -manifolds:

Fernández-Gray ('83), Bryant ('87), Joyce ('96), Hitchin ('00), Grigorian-Yau ('08), Grigorian ('09), ...

► Kovalev's Twisted Connected Sum G_2 -manifolds:

Kovalev ('00), Corti-Haskins-Nordström-Pacini ('12)&('12), Crowley-Nordström ('12), Haskin-Hein-Nordström ('12), ...

G_2 -manifolds

e.g., Joyce ('96)

- Compact real 7d Riemannian manifolds (Y, g)

- ▶ Ricci flat metrics with G_2 holonomy
- ▶ Equivalently: 7d manifold (Y, φ) with torsion-free G_2 -structure φ

$$d\varphi = 0 \quad d *_{g(\varphi)} \varphi = 0$$

- Moduli space M of G_2 manifolds:

- ▶ local properties of G_2 moduli spaces

$$\dim_{\mathbb{R}} M = b_3 : \quad \varphi \rightarrow \varphi + \delta\varphi \quad \text{with} \quad \delta\varphi \quad \text{harmonic}$$

- ▶ global moduli space: no “Calabi-Yau theorem”

$$\text{global map} \quad \mathcal{P} : M \rightarrow H^3(Y) \quad \text{unknown}$$

4d Effective Theory

Beasley-Witten ('02), ...

○ Spectrum of M-theory compactifications

| Multiplicity | harmonic forms | N=1 4d multiplets |
|--------------|----------------|--------------------------|
| 1 | 1 | gravity multiplet |
| $b_3(Y)$ | ρ | chiral multiplets ϕ |
| $b_2(Y)$ | ω | vector multiplets V |

○ Semi-classical 4d N=1 effective action:

- ▶ Chiral coordinates & Kähler potential:

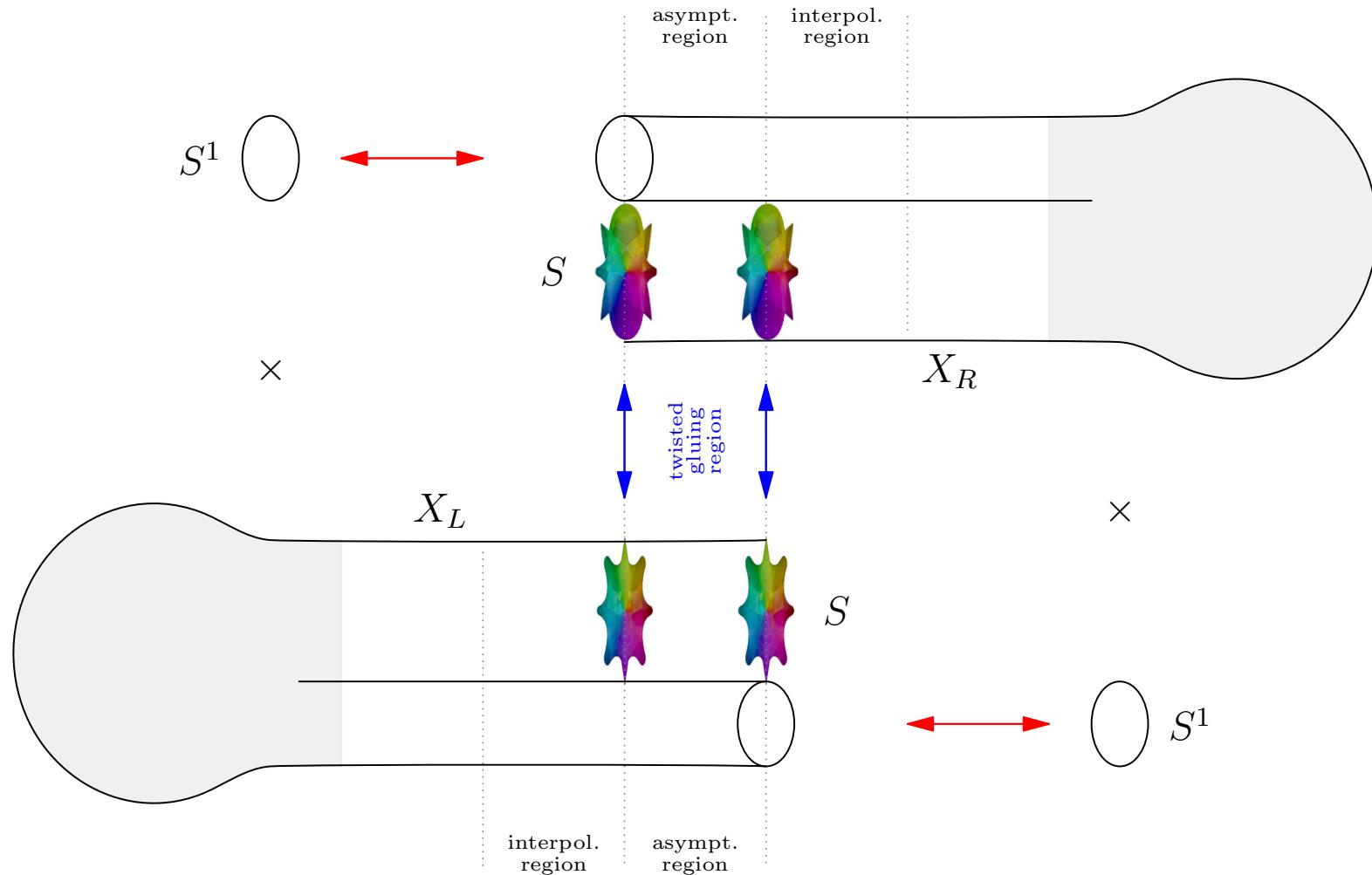
$$C + i\varphi = \Phi \rho \quad K(\phi, \bar{\phi}) = -3 \log \left(\frac{1}{7} \int_Y \varphi \wedge *_{g_\varphi} \varphi \right)$$

- ▶ Superpotential & gauge kinetic coupling functions

$$W(\phi) = \int_Y G \wedge (C + i\varphi) \quad f(\phi) = \phi \int_Y \omega \wedge \omega \wedge \rho$$

Kovalev's Construction

Kovalev ('02), Corti-Haskins-Nordström-Pacini ('12)



- ▶ Asymptotically cylindrical CY 3-folds X_L & X_R have $SU(3)$ holonomy
- ▶ twisted gluing yields G_2 manifold $Y = (X_L \times S^1) \cup (X_R \times S^1)$

Asymptotically Cylindrical CY₃

Corti-Haskins-Nordström-Pacini ('12)

○ Construct asymptotically cylindrical CY 3-folds

- ▶ smooth curve in weak Fano 3-fold P:
i.e., anti-canonical bundle $-K_P$ is nef & big

$$\mathcal{C} = \{s_0 = 0\} \cap \{s_1 = 0\} \quad s_{0/1} \in \Gamma(-K_P)$$

smooth curve in P generic sections

- ▶ K3 fibration from base-point free pencil via blow-up:

$$Z = \text{Bl}_{\mathcal{C}} P = \{(x, \alpha) \in P \times \mathbb{P}^1 \mid \alpha_0 s_0 + \alpha_1 s_1 = 0\} \quad \pi : Z \rightarrow \mathbb{P}^1$$

Blow-up along C K3 fibration

- ▶ Thm [Corti et. al.]:
 $X = Z \setminus \pi^{-1}([0:1])$ admits asymptotically cylindrical CY 3-fold metric

○ Asymptotically CY 3-fold as building block (Z, S)

Matching Condition

Kovalev ('02), Corti-Haskins-Nordström-Pacini ('12), Crowley-Nordström ('12)

○ Matching condition in Kovalev's construction

- ▷ Hyper-Kähler rotation of polarized K3 surfaces

$$\omega_L = \text{Re}(\Omega_R^{2,0}) \quad \Omega_L^{2,0} = \omega_R - i \text{Im}(\Omega_R^{2,0})$$

○ G2-manifolds via orthogonal gluing

- ▷ Torelli's theorem to match polarized K3 surfaces $S_{L/R}$
- ▷ Semi-Fano 3-folds: Application of Beauville's theorem
- S_L & S_R matching $\implies (S_L, Z_L)$ & (S_R, Z_R) matching
- ▷ Gluing of matching building blocks:

$$Y = (Z_L, S_L) \perp (Z_R, S_R)$$

○ Classes of semi-Fano 3-folds

$$\{ \text{weak Fano 3-folds} \} \supset \{ \text{semi Fano 3-folds} \} \supset \{ \text{Fano 3-folds} \}$$

gluing procedure
not addressed/solved yet

Kasprzyk classification
of toric semi-Fano 3-folds

Mori–Mukai classification

4d Effective Theory

Guio-Klemm-Yeh-HJ ('17)

○ semi-classical chiral moduli in the Kovalev limit

- ▶ Volume modulus ν
- ▶ Gluing modulus – Kovalevton κ
- ▶ further moduli fields S

○ semi-classical Kähler potential

$$K = -4 \log(\nu + \bar{\nu}) - 3 \log(\kappa + \bar{\kappa}) - 3 \log(V_{K3}(S))$$

universal moduli
dependence

non-universal moduli
dependence

○ U(1) vector multiplets

Kovalev ('02), ...

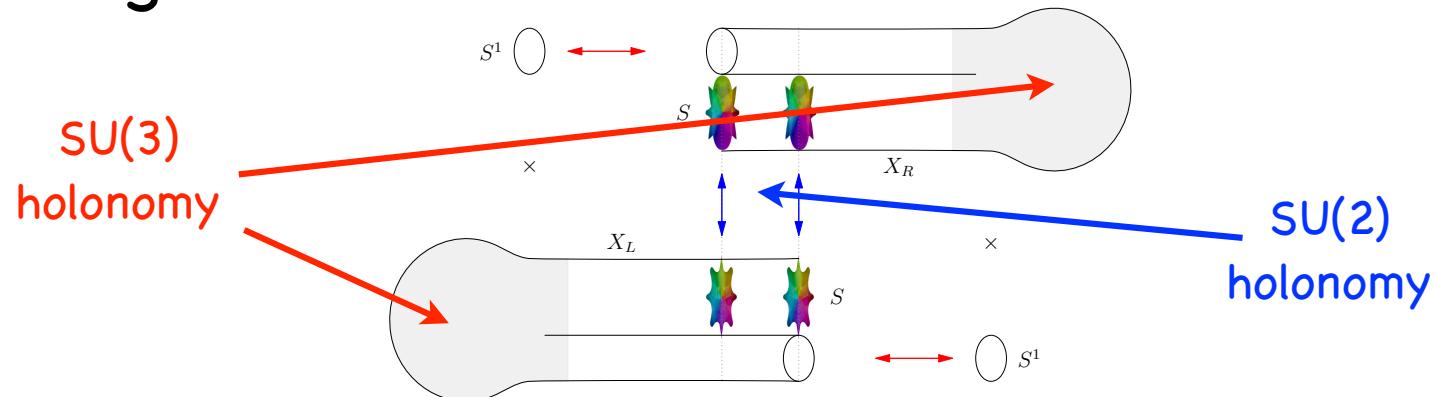
$$b_2(Y) = k_L + k_R + \dim N_L \cap N_R$$

U(1) factors
from X_L & X_R

U(1) factors
from K3 surface S

Gauge Theory Sectors

○ Local geometric structure



○ Gauge theory sectors in the strict Kovalev limit

| local geometry (Kovalev limit) | multiplicity of $\mathcal{N} = 1$ multiplets | | $U(1)$ vector multiplets | |
|--|--|-----------------------------|--------------------------|-------------------|
| | $U(1)$ vectors | chirals | multiplicity | supersym. |
| $Y_L = S_L^1 \times X_L$ $SU(3)$ holonomy | $\dim k_L$ | $\dim k_L$ | $\dim k_L$ | $\mathcal{N} = 2$ |
| $Y_R = S_R^1 \times X_R$ $SU(3)$ holonomy | $\dim k_R$ | $\dim k_R$ | $\dim k_R$ | $\mathcal{N} = 2$ |
| $T^2 \times S \times (0, 1)$ $SU(2)$ holonomy | $\dim N_L \cap N_R$ | $3 \cdot \dim N_L \cap N_R$ | $\dim N_L \cap N_R$ | $\mathcal{N} = 4$ |

Gauge Sector Building Blocks

Morrison-Halverson ('12), Guio-Klemm-Yeh-HJ ('17)

○ Non-generic anti-canonical sections of 3-folds P

- ▶ Factorization of $-K_P$ in line bundles with global sections

$$-K_P = \mathcal{L}_1^{\otimes k_1} \otimes \cdots \otimes \mathcal{L}_s^{\otimes k_s} \quad \text{with section} \quad s_0 = s_{0,1}^{k_1} \cdots s_{0,s}^{k_s}$$

○ Singular blown-up 3-fold

$$Z_{\text{sing}} = \text{Bl}_{\mathcal{C}} P = \left\{ (x, \alpha) \in P \times \mathbb{P}^1 \mid \alpha_0 s_{0,1}^{k_1} \cdots s_{0,s}^{k_s} + \alpha_1 s_1 = 0 \right\}$$

- ▶ A_{k_i} singularities along curves $C_i = \{s_{0,i} = s_1 = \alpha_1 = 0\}$

○ Gauge group

$$\begin{aligned} G &= U(k_1) \times \cdots \times U(k_s)/U_{\text{Diag}}(1) \\ &\simeq SU(k_1) \times \cdots \times SU(k_s) \times U(1)^{s-1} \end{aligned}$$

Matter Spectrum

Guio-Klemm-Yeh-HJ ('17)

| Multiplicity | $\mathcal{N} = 2$ multiplets | | $\mathcal{N} = 1$ multiplets | |
|---|---|------------------|--|----------------------------|
| | G reps. | multiplet | G reps. | multiplet |
| $s - 1$ | 1 | $U(1)$ vector | 1 1 | $U(1)$ vector chiral |
| $i = 1, \dots, s$ | adj $_{SU(k_i)}$ | $SU(k_i)$ vector | adj $_{SU(k_i)}$ adj $_{SU(k_i)}$ | $SU(k_i)$ vector chiral |
| $g(\mathcal{C}_i)$ $1 \leq i \leq s$ | adj $_{SU(k_i)}$ | hyper | adj $_{SU(k_i)}$ adj $_{SU(k_i)}$ | chiral chiral |
| χ_{ij} $1 \leq i < j < s$ | $(\mathbf{k_i}, \mathbf{k_j})_{(+1_i, +1_j)}$ | hyper | $(\mathbf{k_i}, \mathbf{k_j})_{(+1_i, +1_j)}$ $(\bar{\mathbf{k_i}}, \bar{\mathbf{k_j}})_{(-1_i, -1_j)}$ | chiral chiral |
| χ_{is} $1 \leq i < s$ | $(\mathbf{k_i}, \mathbf{k_s})_{(+1_i)}$ | hyper | $(\mathbf{k_i}, \mathbf{k_s})_{(+1_i)}$ $(\bar{\mathbf{k_i}}, \bar{\mathbf{k_s}})_{(-1_i)}$ | chiral chiral |

Gauge Theory Phases

Katz-Morrison-Plesser ('96), Klemm, Mayr ('96), ... , Guio-Klemm-Yeh-HJ ('17)

○ N=2 Higgs & Coulomb branches

- ▷ N=2 Higgs branch:

$$\dim_{\mathbb{C}} H^b = 2 \sum (g(\mathcal{C}_i) - 1) (k_i^2 - 1) + 2 \sum \chi_{ij} k_i k_j - 2(s-1)$$

adjoint d.o.f.
(- 1x SU(k_i) Goldstone hyper.) bifund. d.o.f.

U(1) Goldstone hypers.

- ▷ N=2 Coulomb branch U(1) $^{k_1+\dots+k_s-1}$:

$$\dim_{\mathbb{C}} C^\# = 2 \sum g(\mathcal{C}_i) (k_i - 1) + (k_1 + \dots + k_s - 1)$$

neutral d.o.f. in scalar d.o.f. in
adj. hypers unbroken U(1) vectors

Geometric Phases

○ N=2 Higgs & Coulomb branches

- ▷ Higgs branch – deformation of anti-canonical section s_0 :

$$Z^\flat = \text{Bl}_{\mathcal{C}_{\text{smooth}}} P \quad Y^\flat = (Z_L^\flat, S_L^\sharp) \perp (Z_R, S_R)$$

Higgs G₂ Phase

- ▷ Coulomb branch $U(1)^{k_1+\dots+k_s-1}$ – resolution of A_{k_i} singularities:

$$Z^\sharp = \text{Bl}_{\{\mathcal{C}_1^{k_1}, \dots, \mathcal{C}_s^{k_s}\}} P \quad Y^\sharp = (Z_L^\sharp, S_L^\sharp) \perp (Z_R, S_R)$$

Coulomb G₂ Phase

G_2 Phases

Guio-Klemm-Yeh-HJ ('17)

○ Geometric phases of G_2 -manifolds Y^\flat & $Y^\#$

- ▶ Gauge Theory predicted change in Betti numbers

$$b_2(Y^\flat) = b_2(Y^\#) - (k_1 + \dots + k_s - 1)$$

$$b_3(Y^\flat) = b_3(Y^\#) + \dim_{\mathbb{C}} H^\flat - \dim_{\mathbb{C}} C^\#$$

- ▶ Geometric G_2 phases confirm gauge theory predictions

Harvey-Moore ('99)

○ Semi-classical moduli space expected to be lifted by quantum corrections

Examples I

| s_0 factors | Gauge Group | $\mathcal{N} = 2$ Hypermultiplet spectrum | h^\flat | c^\sharp | b_3^\flat | b_3^\sharp | k^\sharp |
|--|----------------------------------|---|-----------|------------|-------------|--------------|------------|
| $(1, 0)^2(0, 1)^2$ | $SU(2) \times SU(2) \times U(1)$ | $2 \times (\text{adj}, \mathbf{1}); 2 \times (\mathbf{1}, \text{adj}); 4 \times (\mathbf{2}, \mathbf{2})_{+1}$ | 42 | 11 | 50 | 19 | 3 |
| $(1, 0)^2(0, 1)(0, 1)$ | $SU(2) \times U(1)^2$ | $2 \times \text{adj}; 4 \times \mathbf{2}_{(1,0)}; 4 \times \mathbf{2}_{(0,1)}; 2 \times \mathbf{1}_{(1,1)}$ | 38 | 7 | 50 | 19 | 3 |
| $(1, 0)(1, 0)$ $\cdot (0, 1)(0, 1)$ | $U(1)^3$ | $2 \times (1, 1, 0); 4 \times (1, 0, 1); 4 \times (0, 1, 1);$ $4 \times (1, 0, 0); 4 \times (0, 1, 0); 2 \times (0, 0, 1)$ | 34 | 3 | 50 | 19 | 3 |
| $(2, 0)(0, 1)^2$ | $SU(2) \times U(1)$ | $2 \times \text{adj}; 8 \times \mathbf{2}_{+1}$ | 36 | 6 | 50 | 20 | 2 |
| $(2, 0)(0, 1)(0, 1)$ | $U(1)^2$ | $8 \times (1, 0); 8 \times (0, 1); 2 \times (1, 1)$ | 32 | 2 | 50 | 20 | 2 |
| $(1, 1)^2$ | $SU(2)$ | $7 \times \text{adj}$ | 36 | 15 | 50 | 29 | 1 |
| $(1, 1)(1, 1)$ | $U(1)$ | $12 \times (+1)$ | 22 | 1 | 50 | 29 | 1 |
| $(2, 0)(0, 2)$ | $U(1)$ | $16 \times (+1)$ | 30 | 1 | 50 | 21 | 1 |
| $(2, 1)(0, 1)$ | $U(1)$ | $10 \times (+1)$ | 18 | 1 | 50 | 33 | 1 |

Higgs & Coulomb phases of rank 2 Fano 3-fold W_6
 (bi-degree $(1,1)$ hypersurface in $\mathbb{CP}^2 \times \mathbb{CP}^2$)

Examples II

| No. | ρ | Gauge Group | $\mathcal{N} = 2$ Hypermultiplet spectrum | h^\flat | c^\sharp | b_3^\flat | b_3^\sharp | k^\sharp |
|----------------------------|--------|--------------------------------------|---|-----------|------------|-------------|--------------|------------|
| K24, MM34 ₂ | 2 | $SU(3) \times SU(2) \times U(1)$ | $2 \times (\text{adj}, \mathbf{1}); (\mathbf{1}, \text{adj}); 3 \times (\mathbf{3}, \mathbf{2})_{+1}$ | 50 | 14 | 79 | 43 | 4 |
| K32 | 2 | $SU(3)^2 \times U(1)$ | $(\text{adj}, \mathbf{1}); (\mathbf{1}, \text{adj}); 3 \times (\mathbf{3}, \mathbf{3})_{+1}$ | 52 | 13 | 79 | 40 | 5 |
| K35, MM36 ₂ | 2 | $SU(5) \times SU(2) \times U(1)$ | $2 \times (\text{adj}, \mathbf{1}); (\mathbf{5}, \mathbf{2})_{+1}$ | 60 | 22 | 87 | 49 | 6 |
| K36, MM35 ₂ | 2 | $SU(4) \times SU(2) \times U(1)$ | $2 \times (\text{adj}, \mathbf{1}); 2 \times (\mathbf{4}, \mathbf{2})_{+1}$ | 54 | 17 | 81 | 44 | 5 |
| K37, MM33 ₂ | 2 | $SU(4) \times SU(3) \times U(1)$ | $(\text{adj}, \mathbf{1}); 3 \times (\mathbf{4}, \mathbf{3})_{+1}$ | 54 | 12 | 79 | 37 | 6 |
| K62, MM27 ₃ | 3 | $SU(2)^3 \times U(1)^2$ | $(\text{adj}, \mathbf{1}^2); (\mathbf{1}, \text{adj}, \mathbf{1}); (\mathbf{1}^2, \text{adj}); 2 \times (\mathbf{2}^2, \mathbf{1})_{(1,1)}$ $2 \times (\mathbf{2}, \mathbf{1}, \mathbf{2})_{(1,0)}; 2 \times (\mathbf{1}, \mathbf{2}^2)_{(0,1)}$ | 44 | 11 | 73 | 40 | 5 |
| K68, MM25 ₃ | 3 | $SU(3) \times SU(2) \times U(1)^2$ | $(\text{adj}, \mathbf{1}); 3 \times (\mathbf{3}, \mathbf{2})_{(1,1)}; 2 \times (\mathbf{3}, \mathbf{1})_{(1,0)}; (\mathbf{1}, \mathbf{2})_{(0,1)}$ | 42 | 9 | 69 | 36 | 6 |
| K105, MM31 ₃ | 3 | $SU(3)^2 \times SU(2) \times U(1)^2$ | $(\text{adj}, \mathbf{1}^2); (\mathbf{1}, \text{adj}, \mathbf{1}); 2 \times (\mathbf{3}^2, \mathbf{1})_{(1,1)}; (\mathbf{3}, \mathbf{1}, \mathbf{2})_{(1,0)}; (\mathbf{1}, \mathbf{3}, \mathbf{2})_{(0,1)}$ | 50 | 15 | 77 | 42 | 7 |
| K124 | 3 | $SU(4) \times SU(2)^2 \times U(1)^2$ | $(\text{adj}, \mathbf{1}, \mathbf{1}); 2 \times (\mathbf{4}, \mathbf{2}, \mathbf{1})_{(1,1)}; 2 \times (\mathbf{4}, \mathbf{1}, \mathbf{2})_{(1,0)}$ | 48 | 13 | 73 | 38 | 7 |

toric semi-Fano 3-folds of rank 2 & 3 (Kasprzyk list)

Conclusions & Outlook

○ Kovalev's Construction: Explicit G_2 -manifolds

- ▶ Kovalev limit: Explicit effective action derivation
- ▶ Algebraic geometry techniques applicable
(i.e., intersection theory, variation of Hodge structure,...)

○ Proposal: Transitions among G_2 -manifolds

- ▶ Non-Abelian gauge theory sectors with matter
- ▶ Conjecture of geometric transitions among G_2 -manifolds

○ Outlook:

- ▶ Generalization of orthogonal gluing & Kovalev's construction
- ▶ Chiral N=1 spectrum
- ▶ Beyond semi-Fano building blocks (unhiggsable clusters)
- ▶ Non-perturbative Superpotentials from M2-brane Instantons