Bootstrapping $\mathcal{N} \geqslant 2$ SCFTs

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Based on:
1312.5344 w/ C. Beem, P. Liendo, W. Peelaers, L. Rastelli and B. van Rees
1511.07449 w/ P. Liendo
1702.05101 w/ M. Cornagliotto and V. Schomerus

Outline

1. **The (Super)conformal Bootstrap Program**
   - Conformal bootstrap
   - Superconformal bootstrap
2. **A solvable subsector**
3. **Constraining the space of $\mathcal{N}=2$ SCFTs**
4. **$4d$ $\mathcal{N}=3$ SCFTs**
5. **Summary and Outlook**

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4. $4d$ $\mathcal{N} = 3$ **SCFTs**
5. **Summary and Outlook**

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Can we bootstrap specific theories?

→ Particularly helpful if theory is uniquely fixed by a set of discrete data

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Conformal field theory defined by

Set of *local* operators and their correlation functions

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CFT data

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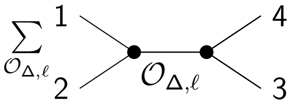
CFT data strongly constrained

- Unitarity
- Associativity of the operator product algebra

Conformal Bootstrap

Crossing Symmetry

$$\langle (\mathcal{O}_1(x_1)\ \mathcal{O}_2(x_2))\mathcal{O}_3(x_3)\ \mathcal{O}_4(x_4)\rangle =$$



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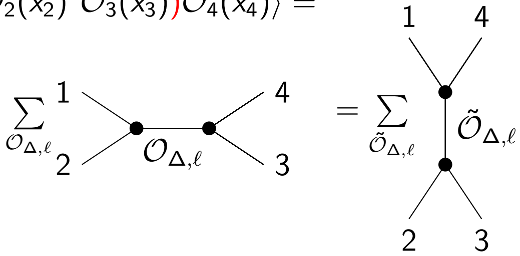

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$\rightarrow$ Yes, for $4d$ $\mathcal{N} \geqslant 2$ [Beem ML Liendo Peelaers Rastelli van Rees]

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Q: Is there a solvable truncation of the crossing equations?

$\rightarrow$ Yes, for $4d$ $\mathcal{N} \geqslant 2$ [Beem ML Liendo Peelaers Rastelli van Rees]
$6d$ $\mathcal{N} = (2,0)$ and $2d$ $\mathcal{N} = (0,4)$ [Beem Rastelli van Rees]

- Step 1: Solve this subsector
- (Step 2: Full blown numerics for the rest)

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Organize operators in representations of superconformal algebra

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- $\rightarrow$ Meromorphic!

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$\hookrightarrow$ anti-holomorphic dependence drops out

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- How much information can we recover from the VOA?

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$\rightarrow$ Stress tensor $T_{\mu\nu}$

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$\hookrightarrow$ Virasoro representations seem to mix different types of $4d$ multiplets

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$\rightarrow$ ...

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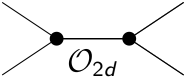
$\sum_{\mathcal{O}_{2d}} \lambda^2_{\mathcal{O}_{2d}}$ 

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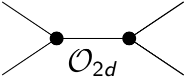
$\sum_{\mathcal{O}_{2d}} \lambda^2_{\mathcal{O}_{2d}}$ 

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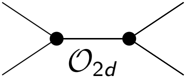
$\sum_{\mathcal{O}_{2d}} \lambda^2_{\mathcal{O}_{2d}}$ 

$\rightarrow$ $\lambda^2_{\mathcal{O}_{2d}} \rightsquigarrow \lambda^2_{\mathcal{O}_{4d}}$

Interpret as four-dimensional quantities

(with some assumptions: interacting theory, unique stress tensor)

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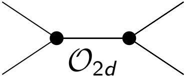
$\sum_{\mathcal{O}_{2d}} \lambda^2_{\mathcal{O}_{2d}}$ 

$\rightarrow \lambda^2_{\mathcal{O}_{2d}} \rightsquigarrow \lambda^2_{\mathcal{O}_{4d}} \underbrace{\geqslant}_{4d \text{ unitarity}} 0$

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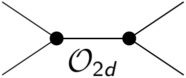


$$\rightarrow \lambda^2_{\mathcal{O}_{2d}} \rightsquigarrow \lambda^2_{\mathcal{O}_{4d}} \underbrace{\geqslant}_{4d \text{ unitarity}} 0 \Rightarrow \text{New unitarity bounds}$$

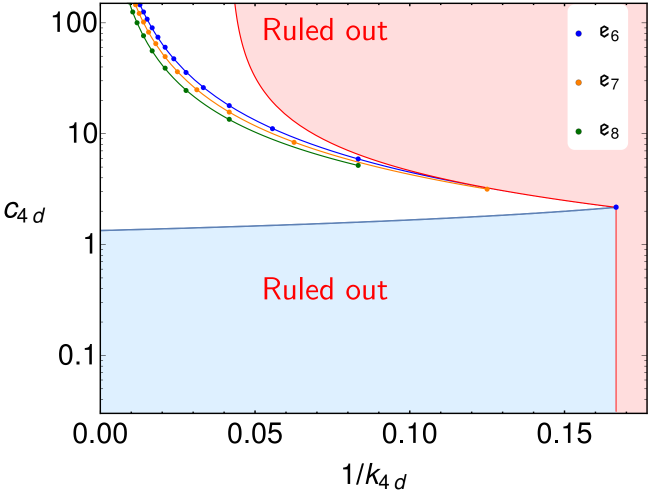
Interpret as four-dimensional quantities

(with some assumptions: interacting theory, unique stress tensor)

Outline

$4d\ \mathcal{N}=2$ SCFTs with E_6 flavor symmetry

$\langle TTTT \rangle$, $\langle J^a J^b J^c J^d \rangle$, $\langle TTJ^a J^b \rangle$



[Beem ML Liendo Peelaers Rastelli van Rees, ML Liendo]

Outline

1. **The (Super)conformal Bootstrap Program**
   - Conformal bootstrap
   - Superconformal bootstrap
2. **A solvable subsector**
3. **Constraining the space of $\mathcal{N}=2$ SCFTs**
4. **$4d$ $\mathcal{N}=3$ SCFTs**
5. **Summary and Outlook**

$\mathcal{N} = 3$ Chiral algebra

- $4d$ $\mathcal{N} \geqslant 3$: some of the extra supercharges commute with $\mathbb{Q}$

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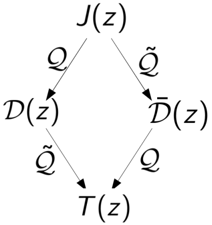

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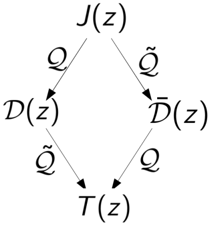

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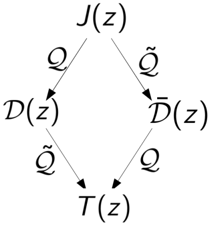

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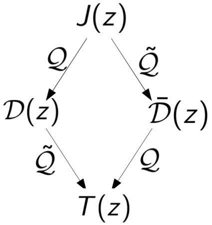

Space of $\mathcal{N} = 3$ SCFTs

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Summary and Outlook

New constraints on the space of allowed $\mathcal{N} = 2, 3$ SCFTs

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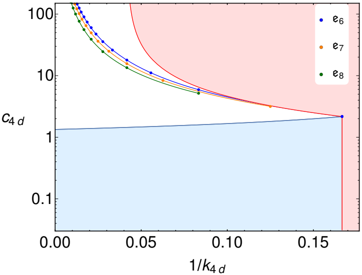

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Numerically solving theories?

- This mixed correlator seems like a good starting point

Thank you!

Backup slides

Outline

Numerical conformal Bootstrap review

Chiral algebra

$4d$ $\mathcal{N} = 3$ SCFTs

Solving $\mathcal{N} = 3$ SCFTs?

Conformal Bootstrap

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[Rattazzi Rychkov Tonni Vichi]

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Sum rule: identical scalars ϕ

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Sum rule: identical scalars ϕ

$\rightarrow$ Identity operator $\lambda_{\mathcal{O}\mathcal{O}\mathbb{1}} = 1$

$$1 = \sum_{\substack{\mathcal{O}_{\Delta_\ell} \neq \mathbb{1} \\ \mathcal{O} \in \phi\phi}} \lambda^2_{\phi\phi\mathcal{O}} \underbrace{\frac{u^{\Delta_\phi} g_{\Delta,\ell}(v,u) - v^{\Delta_\phi} g_{\Delta,\ell}(u,v)}{v^{\Delta_\phi} - u^{\Delta_\phi}}}_{F_{\Delta,\ell}}$$

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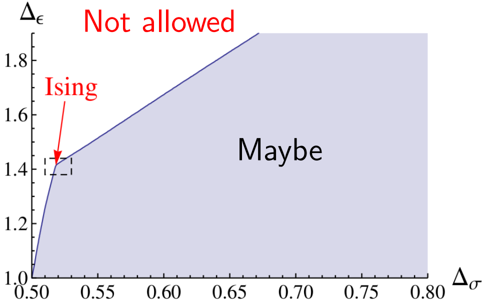
$\rightarrow$ Always true bounds

$3d$ Ising Model

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi, PRD 86 025022]



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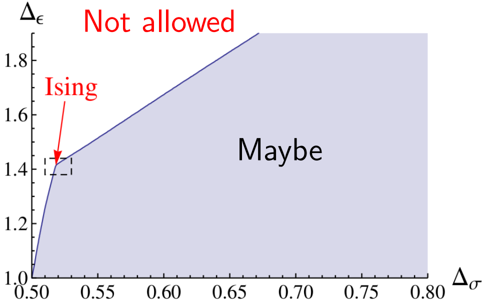

$\rightarrow$ Saturated by $3d$ Ising model

3d Ising Model

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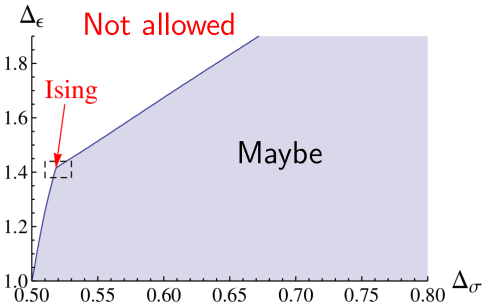

$\longrightarrow$ Saturated by 3d Ising model

$\longrightarrow$ 3d Ising lives at "kink"

Outline

Numerical conformal Bootstrap review

Chiral algebra

$4d$ $\mathcal{N} = 3$ SCFTs

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Chiral algebra

Example: free hypermultiplet

Complex scalars in hypermultiplet are in the cohomology

Chiral algebra

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$$Q^I = \begin{bmatrix} Q \\ \tilde{Q}^\star \end{bmatrix}, \qquad \tilde{Q}^I = \begin{bmatrix} \tilde{Q} \\ -Q^\star \end{bmatrix}$$

Chiral algebra

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Complex scalars in hypermultiplet are in the cohomology

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Chiral algebra

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$$\begin{aligned} u_I &= (1, \bar{z}) \\ q(z,\bar{z}) &= u_I Q^I = Q(z,\bar{z}) + \bar{z}\tilde{Q}^\star(z,\bar{z}), \\ \tilde{q}(z,\bar{z}) &= u_I \tilde{Q}^I = \tilde{Q}(z,\bar{z}) - \bar{z} Q^\star(z,\bar{z}) \end{aligned}$$

Chiral algebra

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& u_I = (1, \bar{z}) \\
& q(z,\bar{z}) = u_I Q^I = Q(z,\bar{z}) + \bar{z}\tilde{Q}^\star(z,\bar{z}), \\
& \tilde{q}(z,\bar{z}) = u_I \tilde{Q}^I = \tilde{Q}(z,\bar{z}) - \bar{z}Q^\star(z,\bar{z}) \\
\rightarrow\ & q(z,zb)\tilde{q}(0) \sim \bar{z}\tilde{Q}^\star(z,\bar{z})\tilde{Q}(0) \sim \tfrac{\bar{z}}{z\bar{z}} = \tfrac{1}{z}
\end{aligned}$$

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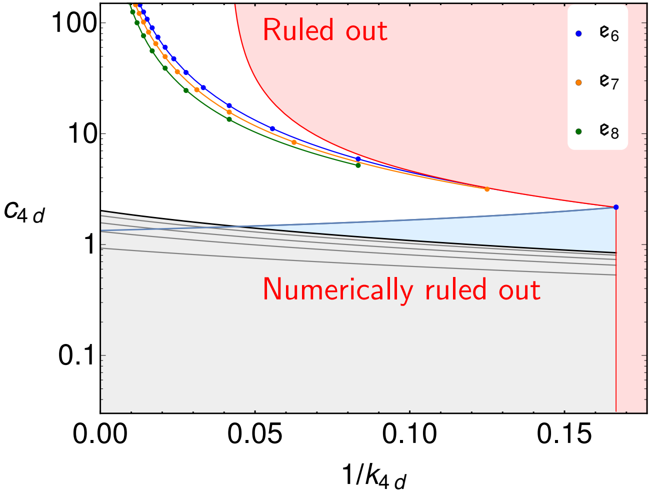
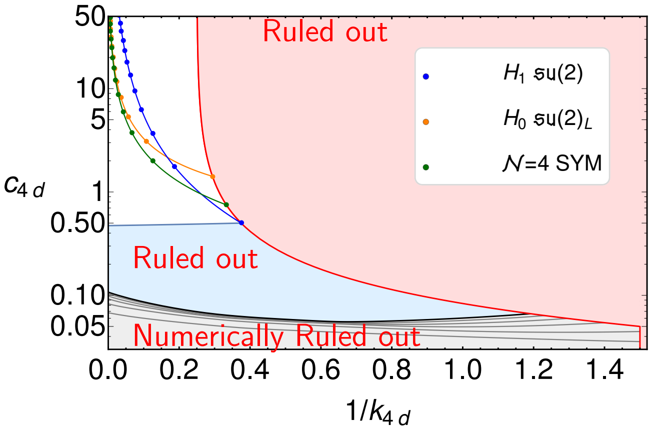


[Beem ML Liendo Rastelli van Rees]

What is the space of consistent SCFTs?

$4d$ $\mathcal{N} = 2$ SCFTs with $SU(2)$ flavor symmetry

$\langle TTTT \rangle , \ \langle J^a J^b J^c J^d \rangle , \ \langle TTJ^a J^b \rangle$



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$\rightarrow$ *Non-trivial* interacting theories

[García-Etxebarria, Regalado] [Aharony, Tachikawa]

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- $\rightarrow$ $SU(3)_R \times U(1)_r$
- $\rightarrow$ No flavor symmetry
- $\rightarrow$ $c = a$
- $\rightarrow$ No exactly marginal deformations
- $\rightarrow$ Just another SCFT

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- $\rightarrow$ No flavor symmetry
- $\rightarrow$ $c = a$
- $\rightarrow$ No exactly marginal deformations
- $\rightarrow$ Just another SCFT

  as $\mathcal{N} = 2$: $SU(2)_R \times U(1)_r \times U(1)_F$

Outline

Numerical conformal Bootstrap review
Chiral algebra
$4d$ $\mathcal{N} = 3$ SCFTs
Solving $\mathcal{N} = 3$ SCFTs?

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    [ML, Liendo, Meneghelli, Mitev]

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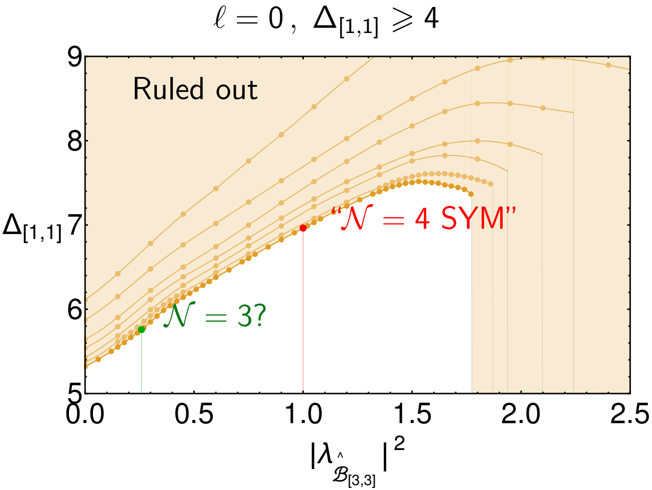
  chiral algebra conjectured by [Nishinaka, Tachikawa]

$4d\ \mathcal{N} = 3$ SCFT with $c = \frac{15}{12}$



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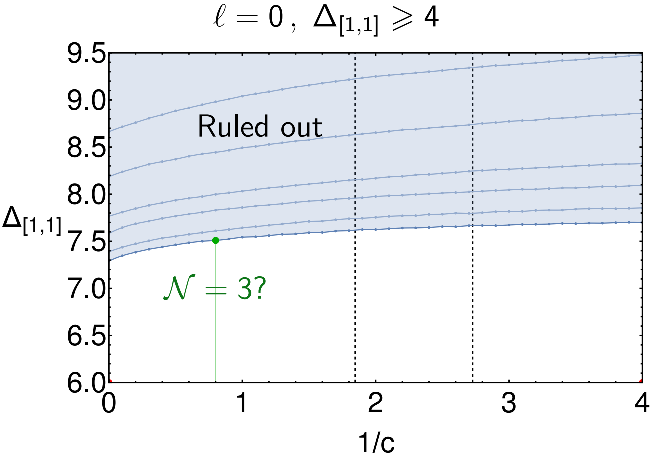

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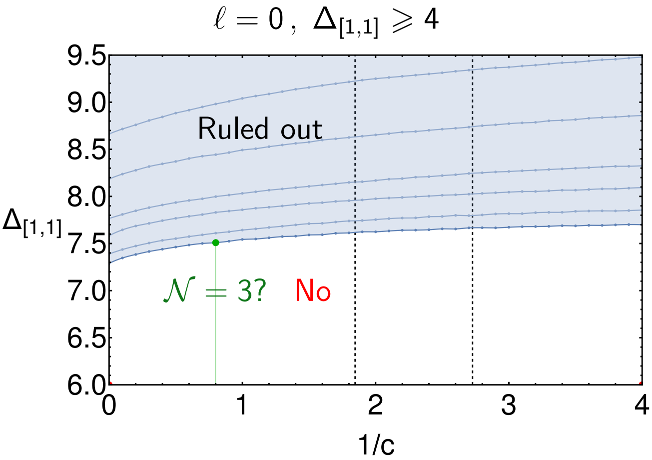

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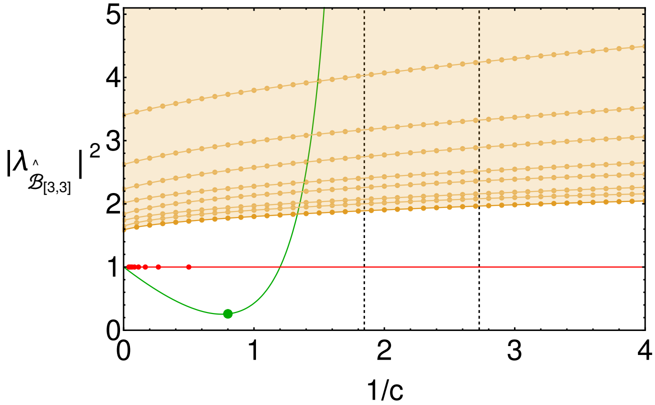

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