Bootstrapping \( \mathcal{N} \geq 2 \) SCFTs

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Based on:
1312.5344 w/ C. Beem, P. Liendo, W. Peelaers, L. Rastelli and B. van Rees
1511.07449 w/ P. Liendo
1702.05101 w/ M. Cornaglioitto and V. Schomerus
1. The (Super)conformal Bootstrap Program
   - Conformal bootstrap
   - Superconformal bootstrap

2. A solvable subsector

3. Constraining the space of $\mathcal{N} = 2$ SCFTs

4. 4d $\mathcal{N} = 3$ SCFTs

5. Summary and Outlook
Outline

1 The (Super)conformal Bootstrap Program
   Conformal bootstrap
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3 Constraining the space of $\mathcal{N} = 2$ SCFTs

4 4d $\mathcal{N} = 3$ SCFTs

5 Summary and Outlook
The (Super)conformal Bootstrap Program

What is the space of consistent (S)CFTs?

- Maximally supersymmetric theories: well known list of theories
  - \( N = 3 \) theories: not known to exist until Garcia-Etxebarria and Regalado
  - \( N = 2 \) theories: large known list of theories many lacking a Lagrangian description

Can we bootstrap specific theories?
  - Particularly helpful if theory is uniquely fixed by a set of discrete data
The (Super)conformal Bootstrap Program

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5 Summary and Outlook
Conformal field theory defined by

Set of *local* operators and their correlation functions
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Set of local operators and their correlation functions

CFT data
\{\mathcal{O}_{\Delta, \ell, \ldots}(x)\}
and
Conformal field theory defined by
Set of *local* operators and their correlation functions

**CFT data**
\[ \{ \mathcal{O}_{\Delta, \ell, \ldots}(x) \} \] and

**Operator Product Expansion**
\[ \mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_k \lambda_{\mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_k} \mathcal{O}_k(0) \]
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**CFT data strongly constrained**

- Unitarity
- Associativity of the operator product algebra
Conformal Bootstrap

Crossing Symmetry

\[ \langle (\mathcal{O}_1(x_1) \mathcal{O}_2(x_2)) \mathcal{O}_3(x_3) \mathcal{O}_4(x_4) \rangle = \]

\[ \sum_{\mathcal{O}_{\Delta,\ell}} \frac{1}{\Delta} \frac{1}{\ell} \]

\[ = \sum_{\tilde{\mathcal{O}}_{\Delta,\ell}} \]

\[ \frac{7}{26} \]
Conformal Bootstrap

Crossing Symmetry

\[
\langle \mathcal{O}_1(x_1) (\mathcal{O}_2(x_2) \mathcal{O}_3(x_3)) \mathcal{O}_4(x_4) \rangle = \sum_{\mathcal{O}_{\Delta,\ell}} 1 \mathcal{O}_{\Delta,\ell}, \ell^4 = \sum_{\tilde{\mathcal{O}}_{\Delta,\ell}} \tilde{\mathcal{O}}_{\Delta,\ell}
\]
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The Superconformal Bootstrap

- Various conformal families related by action of supercharges

Q: Is there a solvable truncation of the crossing equations?
→ Yes, for $4d N \geq 2$ [Beem ML Liendo Peelaers Rastelli van Rees]

$6d N = (2, 0)$ and $2d N = (0, 4)$ [Beem Rastelli van Rees]

Step 1: Solve this subsector
Step 2: Full blown numerics for the rest
Various conformal families related by action of supercharges

Finite re-organization of an infinite amount of data
The Superconformal Bootstrap

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Organize operators in representations of superconformal algebra

\[ \{ \mathcal{O}_{\Delta,(j_1,j_2)}, \} \]
Chiral algebra

Organize operators in representations of superconformal algebra

$$\{ \mathcal{O}_{\Delta,(j_1,j_2)}, R, r, f \}$$
Chiral algebra

Organize operators in representations of superconformal algebra

\( \{ \mathcal{O}_{\Delta,(j_1,j_2)}, R^{\pm}, r^{\pm}, f \} \)

Claim

→ Pick a plane \( \mathbb{R}^2 \in \mathbb{R}^4, (z, \bar{z}) \in \mathbb{R}^2 \)
Chiral algebra

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\[ \{ \mathcal{O}_{\Delta,(j_1,j_2)}, R, r, f \} \]

Claim

→ Pick a plane \( \mathbb{R}^2 \in \mathbb{R}^4, (z, \bar{z}) \in \mathbb{R}^2 \)
→ Restrict to operators with \( \Delta = 2R + j_1 + j_2 \)
Chiral algebra

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\[ \{ \mathcal{O}_{\Delta, \{j_1, j_2\}, r, f} \} \]

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\[ \langle \mathcal{O}_1^{l_1}(z_1, \bar{z}_1) \ldots \mathcal{O}_n^{l_n}(z_n, \bar{z}_n) \rangle \]
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\[ u_{l_1}(\bar{z}_1) \ldots u_{l_n}(\bar{z}_n) \langle \mathcal{O}_{1}^{l_1}(z_1, \bar{z}_1) \ldots \mathcal{O}_{n}^{l_n}(z_n, \bar{z}_n) \rangle \]
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\[
u_{l_1}(\bar{z}_1) \ldots u_{l_n}(\bar{z}_n)\langle O_{1}^{l_1}(z_1, \bar{z}_1) \ldots O_{n}^{l_n}(z_n, \bar{z}_n) \rangle = f(z_i)\]

→ Meromorphic!
Why?

- Subsector = Cohomology of nilpotent $\mathbb{Q}$
Why?

- Subsector = Cohomology of nilpotent $Q \sim Q + S$
Chiral algebra

Why?

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\[ \mathfrak{sl}_2 \times \mathfrak{sl}_2 \]
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- $\Rightarrow$ twisted translations $u_I(\bar{z})$
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- Diagonal subalgebra $\mathfrak{sl}_2 \times \mathfrak{su}(2)_R$ is $\mathbb{Q}$ exact
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- twisted translations $u_I(\bar{z})$
- diagonal subalgebra $\bar{\mathfrak{sl}}_2 \times \mathfrak{su}(2)_R$ is $Q$ exact
- anti-holomorphic dependence drops out
$4d \mathcal{N} \geq 2$ SCFT $\rightarrow$ VOA
\[ 4d \mathcal{N} \geq 2 \text{ SCFT} \rightarrow \text{ VOA} \]

→ Cohomology classes ⇒ Vertex operators
\[ 4d \mathcal{N} \geq 2 \text{ SCFT} \rightarrow \text{VOA} \]

\[ \rightarrow \text{Cohomology classes} \Rightarrow \text{Vertex operators} \]
\[ \rightarrow \text{conformal weight } h = R + j_1 + j_2 \geq 0 \]
$4d \mathcal{N} \geq 2$ SCFT $\rightarrow$ VOA

→ Cohomology classes $\Rightarrow$ Vertex operators
→ conformal weight $h = R + j_1 + j_2 \geq 0$
→ Each $\mathcal{N} = 2$ multiplet contributes at most with one $\mathfrak{sl}_2$ primary
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$\quad$ ▶ Obtain VOA from 4d $\mathcal{N} = 2$ SCFT
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▶ Given a VOA does there exist a 4d SCFT?
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$4d \ \mathcal{N} \geq 2 \ \text{SCFT} \longrightarrow \ \text{VOA}$

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- Obtain VOA from $4d \ \mathcal{N} = 2 \ \text{SCFT}$
- Given a VOA does there exist a $4d$ SCFT?
  - Give an example of what can go wrong
- How much information can we recover from the VOA?
Which operators are in the cohomology?

→ Stress tensor $T_{\mu\nu}$
Which operators are in the cohomology?

→ Stress tensor $T_{\mu\nu} \rightsquigarrow$ superdescendant
4d $\mathcal{N} \geq 2$ SCFT $\longrightarrow$ VOA

Which operators are in the cohomology?

$\rightarrow$ Stress tensor $T_{\mu\nu} \rightsquigarrow$ superdescendant
$\rightarrow$ Stress tensor supermultiplet
Which operators are in the cohomology?

→ Stress tensor $T_{\mu\nu} \leadsto$ superdescendant

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$$T(z)T(0) \sim -12 \frac{c_{4d}/2}{z^4} + 2 \frac{T(0)}{z^2} + \frac{\partial T(0)}{z} + \ldots ,$$
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↔ $c_{2d} = -12c_{4d}$
↔ Virasoro representations seem to mix different types of 4d multiplets
Which operators are in the cohomology?

→ Flavor symmetries current multiplet
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↔ Affine Kac Moody current algebra

\[
J^a(z)J^b(0) \sim -\frac{k_{4d}/2\delta^{ab}}{z^2} + if^{abc}J^c(0) + \ldots,
\]
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← \( k_{2d} = -\frac{k_{4d}}{2} \)
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→ \ldots
What is the space of consistent SCFTs?

$4d \mathcal{N} = 2$ SCFTs with a flavor symmetry

$\langle TTTT \rangle, \langle J^a J^b J^c J^d \rangle, \langle TTJ^a J^b \rangle$
What is the space of consistent SCFTs?

$4d \mathcal{N} = 2$ SCFTs with a flavor symmetry

$\langle TTTT \rangle, \langle J^a J^b J^c J^d \rangle, \langle TTJ^a J^b \rangle$ functions of $c_{2d}$ and $k_{2d}$
What is the space of consistent SCFTs?

4d $\mathcal{N} = 2$ SCFTs with a flavor symmetry $\langle TTTT \rangle$, $\langle J^a J^b J^c J^d \rangle$, $\langle T T J^a J^b \rangle$ functions of $c_{2d}$ and $k_{2d}$

- Block decomposition:

$$\sum \mathcal{O}_{2d} \lambda_{\mathcal{O}_{2d}}^2 \longrightarrow \mathcal{O}_{2d}$$
What is the space of consistent SCFTs?

$4d \mathcal{N} = 2$ SCFTs with a flavor symmetry
\[ \langle TTTT \rangle, \langle J^a J^b J^c J^d \rangle, \langle TT J^a J^b \rangle \text{ functions of } c_{2d} \text{ and } k_{2d} \]

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$$\sum_{\mathcal{O}_{2d}} \lambda_{\mathcal{O}_{2d}}^2 \mathcal{O}_{2d} \quad \rightarrow \quad \lambda_{\mathcal{O}_{2d}}^2 \sim \lambda_{\mathcal{O}_{4d}}^2$$

Interpret as four-dimensional quantities

(with some assumptions: interacting theory, unique stress tensor)
What is the space of consistent SCFTs?

4d $\mathcal{N} = 2$ SCFTs with a flavor symmetry

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$$\sum_{\mathcal{O}_{2d}} \lambda_{\mathcal{O}_{2d}}^2 \quad \Rightarrow \quad \mathcal{O}_{2d}$$

$$\Rightarrow \lambda_{\mathcal{O}_{2d}}^2 \sim \lambda_{\mathcal{O}_{4d}}^2 \geq 0$$

4d unitarity

Interpret as four-dimensional quantities

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What is the space of consistent SCFTs?

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$\langle TTTT \rangle, \langle J^a J^b J^c J^d \rangle, \langle TTJ^a J^b \rangle$ functions of $c_{2d}$ and $k_{2d}$

- Block decomposition:

$$
\sum_{\mathcal{O}_{2d}} \lambda_{\mathcal{O}_{2d}}^2 \rightarrow \lambda_{\mathcal{O}_{2d}}^2 \implies \lambda_{\mathcal{O}_{4d}}^2 \geq 0 \implies \text{New unitarity bounds}
$$

4d unitarity

Interpret as four-dimensional quantities

(with some assumptions: interacting theory, unique stress tensor)
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5. Summary and Outlook
4d $\mathcal{N} = 2$ SCFTs with $E_6$ flavor symmetry

\[ \langle TTTT \rangle, \quad \langle J^a J^b J^c J^d \rangle, \quad \langle TT J^a J^b \rangle \]
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5. Summary and Outlook
$\mathcal{N} = 3$ Chiral algebra

- $4d \mathcal{N} \geq 3$: some of the extra supercharges commute with $\mathcal{Q}$
\[ \mathcal{N} = 3 \] Chiral algebra

- $4d \mathcal{N} \geq 3$: some of the extra supercharges commute with $\mathbb{Q}$
  \[ \rightarrow 4d \mathcal{N} = 4 \Rightarrow 2d \text{ "small" } \mathcal{N} = 4 \text{ chiral algebra} \]
\[ \mathcal{N} = 3 \] Chiral algebra

- \( 4d \mathcal{N} \geq 3 \): some of the extra supercharges commute with \( Q \)
  - \( 4d \mathcal{N} = 4 \) \( \Rightarrow \) 2d “small” \( \mathcal{N} = 4 \) chiral algebra
  - \( 4d \mathcal{N} = 3 \) \( \Rightarrow \) 2d \( \mathcal{N} = 2 \) chiral algebra [Nishinaka, Tachikawa]
$\mathcal{N} = 3$ Chiral algebra

- $4d \mathcal{N} \geq 3$: some of the extra supercharges commute with $Q$
  - $4d \mathcal{N} = 4 \Rightarrow 2d$ "small" $\mathcal{N} = 4$ chiral algebra
  - $4d \mathcal{N} = 3 \Rightarrow 2d \mathcal{N} = 2$ chiral algebra [Nishinaka, Tachikawa]

- $2d$ stress tensor promoted to supermultiplet $\mathcal{J}$

![Diagram](attachment:image.png)
\[ \mathcal{N} = 3 \text{ Chiral algebra} \]

- \(4d \mathcal{N} \geq 3\): some of the extra supercharges commute with \(Q\)
  - \(4d \mathcal{N} = 4 \Rightarrow 2d \) "small" \(\mathcal{N} = 4\) chiral algebra
  - \(4d \mathcal{N} = 3 \Rightarrow 2d \mathcal{N} = 2\) chiral algebra [Nishinaka, Tachikawa]

- \(2d\) stress tensor promoted to supermultiplet \(\mathcal{J}\)

\[ 2d \mathcal{N} = 2 \text{ Stress tensor } \mathcal{J} \]

\[ \rightarrow \text{ A trivial statement in } 2d:\]
$\mathcal{N} = 3$ Chiral algebra

- $4d \mathcal{N} \geq 3$: some of the extra supercharges commute with $Q$
  - $4d \mathcal{N} = 4 \Rightarrow 2d$ “small” $\mathcal{N} = 4$ chiral algebra
  - $4d \mathcal{N} = 3 \Rightarrow 2d \mathcal{N} = 2$ chiral algebra [Nishinaka, Tachikawa]

- $2d$ stress tensor promoted to supermultiplet $\mathcal{J}$

$2d \mathcal{N} = 2$ Stress tensor $\mathcal{J}$

- A trivial statement in $2d$: $\langle \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \rangle$ is fixed in terms of $c_{2d}$
\[ \mathcal{N} = 3 \text{ Chiral algebra} \]

- \( 4d \mathcal{N} \geq 3 \): some of the extra supercharges commute with \( Q \)
  - \( 4d \mathcal{N} = 4 \) \( \Rightarrow \) 2d “small” \( \mathcal{N} = 4 \) chiral algebra
  - \( 4d \mathcal{N} = 3 \) \( \Rightarrow \) 2d \( \mathcal{N} = 2 \) chiral algebra [Nishinaka, Tachikawa]

- 2d stress tensor promoted to supermultiplet \( \mathcal{J} \)

\[ 2d \mathcal{N} = 2 \text{ Stress tensor } \mathcal{J} \]

- A trivial statement in 2d:
  \[ \langle \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \rangle \text{ is fixed in terms of } c_{2d} \]
- Present in any local \( \mathcal{N} = 3 \) SCFT
Space of $\mathcal{N} = 3$ SCFTs

$2d$ $\mathcal{N} = 2$ Stress tensor $\mathcal{J}$

$\langle \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \rangle$ is fixed in terms of $c_{2d} \Rightarrow \lambda_{\mathcal{O}_{2d}}^2 \rightsquigarrow \lambda_{\mathcal{O}_{4d}}^2$
Space of $\mathcal{N} = 3$ SCFTs

$2d \mathcal{N} = 2$ Stress tensor $\mathcal{J}$

$\langle \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \rangle$ is fixed in terms of $c_{2d} \Rightarrow \lambda_{\mathcal{O}_{2d}}^2 \sim \lambda_{\mathcal{O}_{4d}}^2$

$c_{4d} \geq \frac{13}{24}$ [Cornagliotto, ML, Schomerus]

$\hookrightarrow$ Not saturated by any known SCFT

[91x250]Space of $\mathcal{N} = 3$ SCFTs

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$\hookrightarrow$ Similar bounds in $\mathcal{N} = 4$ and $\mathcal{N} = 2$ saturated by known SCFTs [Beem, Rastelli, van Rees] [Liendo, Ramirez, Seo]
Space of $\mathcal{N} = 3$ SCFTs

2d $\mathcal{N} = 2$ Stress tensor $\mathcal{J}$

$\langle \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \rangle$ is fixed in terms of $c_{2d} \Rightarrow \lambda_{O_{2d}}^2 \sim \lambda_{O_{4d}}^2$

\[
c_{4d} \geq \frac{13}{24} \quad \text{[Cornagliotto, ML, Schomerus]}
\]

$\rightarrow$ Not saturated by any known SCFT

$\rightarrow$ Similar bounds in $\mathcal{N} = 4$ and $\mathcal{N} = 2$ saturated by known SCFTs [Beem, Rastelli, van Rees] [Liendo, Ramirez, Seo]

$\rightarrow$ $c_{4d} = \frac{13}{24} \Rightarrow$ reconstruct 4d operators appearing in $\mathcal{J} \mathcal{J}$
Space of $\mathcal{N} = 3$ SCFTs

$2d$ $\mathcal{N} = 2$ Stress tensor $\mathcal{J}$

$\langle \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \rangle$ is fixed in terms of $c_{2d} \Rightarrow \lambda_{\mathcal{O}_{2d}}^2 \sim \lambda_{\mathcal{O}_{4d}}^2$

$$c_{4d} \geq \frac{13}{24} \quad \text{[Cornaglioitto, ML, Schomerus]}$$

$\rightarrow$ Not saturated by any known SCFT

$\rightarrow$ Similar bounds in $\mathcal{N} = 4$ and $\mathcal{N} = 2$ saturated by known SCFTs [Beem, Rastelli, van Rees] [Liendo, Ramirez, Seo]

$\rightarrow$ $c_{4d} = \frac{13}{24} \Rightarrow$ reconstruct $4d$ operators appearing in $\mathcal{J} \mathcal{J}$

$\rightarrow$ Signs of norms inconsistent with an *interacting* $4d$ SCFT existing
Space of $\mathcal{N} = 3$ SCFTs

$2d \mathcal{N} = 2$ Stress tensor $\mathcal{J}$

$\langle \mathcal{J} \mathcal{J} \mathcal{J} \mathcal{J} \rangle$ is fixed in terms of $c_{2d} \Rightarrow \lambda_{\mathcal{O}_{2d}}^2 \leadsto \lambda_{\mathcal{O}_{4d}}^2$

$c_{4d} > \frac{13}{24}$ [Cornaglio, ML, Schomerus]

$\mapsto$ Not saturated by any known SCFT

$\mapsto$ Similar bounds in $\mathcal{N} = 4$ and $\mathcal{N} = 2$ saturated by known SCFTs [Beem, Rastelli, van Rees] [Liendo, Ramirez, Seo]

$\rightarrow$ $c_{4d} = \frac{13}{24} \Rightarrow$ reconstruct $4d$ operators appearing in $\mathcal{J} \mathcal{J}$

$\rightarrow$ Signs of norms inconsistent with an interacting $4d$ SCFT existing
1 The (Super)conformal Bootstrap Program
   Conformal bootstrap
   Superconformal bootstrap

2 A solvable subsector

3 Constraining the space of $\mathcal{N} = 2$ SCFTs

4 $4d$ $\mathcal{N} = 3$ SCFTs

5 Summary and Outlook
New constraints on the space of allowed $\mathcal{N} = 2, 3$ SCFTs
Summary and Outlook

New constraints on the space of allowed $\mathcal{N} = 2, 3$ SCFTs

→ No “minimal” $\mathcal{N} = 3$ SCFT with $c = \frac{13}{24}$
Summary and Outlook

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→ No “minimal” $\mathcal{N} = 3$ SCFT with $c = \frac{13}{24}$

← can we improve on this bound analytically?

What are the conditions for a VOA to correspond to a $4d$ SCFT?
Summary and Outlook

New constraints on the space of allowed $\mathcal{N} = 2, 3$ SCFTs

→ No “minimal” $\mathcal{N} = 3$ SCFT with $c = \frac{13}{24}$
   ← can we improve on this bound analytically?
   What are the conditions for a VOA to correspond to a 4d SCFT?

→ Can the numerical bootstrap complement these?
New constraints on the space of allowed $\mathcal{N} = 2, 3$ SCFTs

→ No “minimal” $\mathcal{N} = 3$ SCFT with $c = \frac{13}{24}$
  
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→ Is $c_{4d}/k_{4d} \geq \ldots$?
Summary and Outlook

New constraint on the space of allowed $\mathcal{N} = 2, 3$ SCFTs

→ No "minimal" $\mathcal{N} = 3$ SCFT with $c = \frac{13}{24}$
  → can we improve on this bound analytically?
  What are the conditions for a VOA to correspond to a 4d SCFT?

→ Can the numerical bootstrap complement these?
→ Is $c_{4d}/k_{4d} \geq ...$?

Numerically solving theories?

▶ This mixed correlator seems like a good starting point
Thank you!
Outline

Numerical conformal Bootstrap review
Chiral algebra
4d $\mathcal{N} = 3$ SCFTs
Solving $\mathcal{N} = 3$ SCFTs?
→ Solve crossing equations for \textit{all} four-point functions
Conformal Bootstrap

→ Solve crossing equations for all four-point functions

[Rattazzi Rychkov Tonni Vichi]

▶ Solving ⇒ constraining
Conformal Bootstrap

→ Solve crossing equations for \textit{all} four-point functions

[Rattazzi Rychkov Tonni Vichi]

> Solving \Rightarrow \textit{constraining}

→ Guess for the spectrum

\[\sum_{O \in \phi \phi} O^{\Delta} \lambda_{O O}^{1} = 1\]
Conformal Bootstrap

→ Solve crossing equations for all four-point functions

[Rattazzi Rychkov Tonni Vichi]

▶ Solving \( \Rightarrow \) constraining
  → Guess for the spectrum
    ← there's a large gap in the spectrum
Conformal Bootstrap

→ Solve crossing equations for all four-point functions

[Rattazzi Rychkov Tonni Vichi]

▶ Solving ⇒ constraining

→ Guess for the spectrum

↔ there’s a large gap in the spectrum

→ Can it ever define a consistent CFT?
Conformal Bootstrap

→ Solve crossing equations for \textit{all} four-point functions

[Rattazzi Rychkov Tonni Vichi]

▷ Solving $\Rightarrow$ constraining
  → Guess for the spectrum
    ← there’s a large gap in the spectrum
  → Can it ever define a consistent CFT?

\textbf{Sum rule: identical scalars}$\phi$

\[\sum_{O}^{\Delta_{\ell} \neq 1} O \phi \phi_{\Delta_{\lambda}^{2}} \left( v, u \right) - v \Delta_{\phi} - u \Delta_{\phi} \]

\[\frac{F_{\Delta_{\lambda}^{3/18}}}{O} \]
Conformal Bootstrap

→ Solve crossing equations for all four-point functions

[Rattazzi Rychkov Tonni Vichi]

▸ Solving ⇒ constraining
  → Guess for the spectrum
    ← there’s a large gap in the spectrum
  → Can it ever define a consistent CFT?

**Sum rule: identical scalars \( \phi \)**

→ Identity operator \( \lambda_{\mathcal{O}\mathcal{O}_1} = 1 \)

\[
1 = \sum_{\substack{\mathcal{O}_{\Delta,\ell} \neq 1 \\ \mathcal{O} \in \phi \phi}} \lambda_{\phi\phi}^2 \frac{u^{\Delta_{\phi}} g_{\Delta,\ell}(v, u) - v^{\Delta_{\phi}} g_{\Delta,\ell}(u, v)}{v^{\Delta_{\phi}} - u^{\Delta_{\phi}}} \frac{F_{\Delta,\ell}}{v^{\Delta_{\phi}} - u^{\Delta_{\phi}}}
\]
Conformal Bootstrap

Sum rule

\[ 1 = \sum_{\mathcal{O}_{\Delta, \ell} \neq \mathbb{1}} \lambda_{\phi \phi}^2 F_{\Delta, \ell} \]

\( \mathcal{O} \in \phi \phi \)

Find Functional \( \Psi \) such that

\( \hat{\psi} \cdot 1 < 0 \) \( \hat{\psi} \cdot F_{\Delta, \ell}(u, v) \geq 0 \) for all \( \{\Delta, \ell\} \) in spectrum

\( \rightarrow \) Spectrum is inconsistent \( \Rightarrow \) rule out CFT

\( \rightarrow \) Truncate \( \hat{\psi} = m, n \leq \Lambda \)

\( \sum_{m, n} a_{mn} \partial^m z \partial^n \bar{z} | z = \bar{z} = \frac{1}{2} \)

\( \rightarrow \) Increase \( \Lambda \) \( \Rightarrow \) bounds get stronger

\( \rightarrow \) Always true bounds
Conformal Bootstrap

Sum rule

$$1 = \sum_{\mathcal{O}} \lambda_{\phi \phi}^2 \mathcal{O} F_{\Delta, \ell}$$

\[ O_{\Delta \ell} \neq \mathbb{1} \]

\[ O \in \phi \phi \]

- Find Functional $\Psi$ such that
  - $\psi \cdot 1 < 0 \ (\mathbb{1})$
  - $\psi \cdot F_{\Delta, \ell}(u, v) \geq 0$ for all $\{\Delta, \ell\}$ in spectrum
Conformal Bootstrap

Sum rule

\[ 1 = \sum_{\mathcal{O} \neq \mathbb{I}, \mathcal{O} \in \mathcal{O}_{\phi \phi}} \lambda_{\phi \phi}^2 \mathcal{O} F_{\Delta, \ell} \]

- Find Functional \( \Psi \) such that
  \[ \psi \cdot \mathbb{1} < 0 \, (\mathbb{I}) \]
  \[ \psi \cdot F_{\Delta, \ell}(u, v) \geq 0 \text{ for all } \{\Delta, \ell\} \text{ in spectrum} \]

- Spectrum is inconsistent \( \Rightarrow \) rule out CFT
Conformal Bootstrap

Sum rule

\[ 1 = \sum_{\Delta, \ell \neq 1, O \in \phi \phi} \lambda_{\phi \phi}^2 O F_{\Delta, \ell} \]

- Find Functional $\Psi$ such that
  \[ \psi \cdot 1 < 0 \quad (1) \]
  \[ \psi \cdot F_{\Delta, \ell}(u, v) \geq 0 \text{ for all } \{\Delta, \ell\} \text{ in spectrum} \]

- Spectrum is inconsistent $\Rightarrow$ rule out CFT

- Truncate

\[ \psi = \sum_{m, n \leq \Lambda} a_{mn} \partial_z^m \partial_{\bar{z}}^n \bigg|_{z = \bar{z} = \frac{1}{2}} \]

- Increase $\Lambda \Rightarrow$ bounds get stronger

- Always true bounds
Conformal Bootstrap

**Sum rule**

\[
1 = \sum_{O \not= \mathbb{1}, O \in \phi \phi} \lambda_{\phi \phi}^2 F_{\Delta, \ell}
\]

- Find Functional $\Psi$ such that
  - $\psi \cdot 1 < 0 (\mathbb{1})$
  - $\psi \cdot F_{\Delta, \ell}(u, v) \geq 0$ for all $\{\Delta, \ell\}$ in spectrum

  → Spectrum is inconsistent $\Rightarrow$ rule out CFT

- Truncate

  \[
  \psi = \sum_{m,n \leq \Lambda} a_{mn} \partial_z^m \partial_{\bar{z}}^n \big|_{z = \bar{z} = \frac{1}{2}}
  \]

  → Increase $\Lambda \Rightarrow$ bounds get stronger
Conformal Bootstrap

**Sum rule**

\[ 1 = \sum_{\mathcal{O} \Delta, \ell \neq 1} \lambda_{\phi\phi}^2 F_{\mathcal{O}, \Delta, \ell} \]

\( \mathcal{O} \in \phi\phi \)

- Find Functional \( \Psi \) such that
  - \( \psi \cdot 1 < 0 \) \( (\mathbb{I}) \)
  - \( \psi \cdot F_{\Delta, \ell}(u, v) \geq 0 \) for all \( \{\Delta, \ell\} \) in spectrum

→ Spectrum is inconsistent \( \Rightarrow \) rule out CFT

- Truncate

\[ \psi = \sum_{m, n \leq \Lambda} a_{mn} \partial_z^m \partial_{\bar{z}}^n \bigg|_{z=\bar{z}=\frac{1}{2}} \]

→ Increase \( \Lambda \) \( \Rightarrow \) bounds get stronger

→ Always true bounds
3d Ising Model

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi, PRD 86 025022]
3d Ising Model

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi, PRD 86 025022]

→ Saturated by 3d Ising model
3d Ising Model

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi, PRD 86 025022]

→ Saturated by 3d Ising model
→ 3d Ising lives at “kink”
Outline

Numerical conformal Bootstrap review
Chiral algebra
$4d \mathcal{N} = 3$ SCFTs
Solving $\mathcal{N} = 3$ SCFTs?
Example: free hypermultiplet

Complex scalars in hypermultiplet are in the cohomology
Chiral algebra

Example: free hypermultiplet

Complex scalars in hypermultiplet are in the cohomology

\[ Q' = \begin{bmatrix} Q \\ \tilde{Q}^* \end{bmatrix}, \quad \tilde{Q}' = \begin{bmatrix} \tilde{Q} \\ -Q^* \end{bmatrix} \]
Chiral algebra

Example: free hypermultiplet

Complex scalars in hypermultiplet are in the cohomology

\[ Q' = \begin{bmatrix} Q \\ \tilde{Q}^* \end{bmatrix}, \quad \tilde{Q}' = \begin{bmatrix} \tilde{Q} \\ -Q^* \end{bmatrix} \]

\[ u_I = (1, \bar{z}) \]
Chiral algebra

Example: free hypermultiplet

Complex scalars in hypermultiplet are in the cohomology

\[ Q' = \begin{bmatrix} Q \\ \tilde{Q}^* \end{bmatrix}, \quad \tilde{Q}' = \begin{bmatrix} \tilde{Q} \\ -Q^* \end{bmatrix} \]

\[ u_I = (1, \bar{z}) \]

\[ q(z, \bar{z}) = u_I Q' = Q(z, \bar{z}) + \bar{z} \tilde{Q}^*(z, \bar{z}), \]

\[ \tilde{q}(z, \bar{z}) = u_I \tilde{Q}' = \tilde{Q}(z, \bar{z}) - \bar{z} Q^*(z, \bar{z}) \]
Chiral algebra

Example: free hypermultiplet

Complex scalars in hypermultiplet are in the cohomology

\[ Q' = \begin{bmatrix} Q \\ \tilde{Q}^* \end{bmatrix}, \quad \tilde{Q}' = \begin{bmatrix} \tilde{Q} \\ -Q^* \end{bmatrix} \]

\[ u_I = (1, \bar{z}) \]
\[ q(z, \bar{z}) = u_I Q' = Q(z, \bar{z}) + \bar{z} \tilde{Q}^*(z, \bar{z}), \]
\[ \tilde{q}(z, \bar{z}) = u_I \tilde{Q}' = \tilde{Q}(z, \bar{z}) - \bar{z} Q^*(z, \bar{z}) \]
\[ \rightarrow q(z, zb)\tilde{q}(0) \sim \bar{z} \tilde{Q}^*(z, \bar{z}) \tilde{Q}(0) \sim \frac{\bar{z}}{z\bar{z}} = \frac{1}{z} \]
$4d\, \mathcal{N} = 2$ SCFTs with $E_6$ flavor symmetry

\[ \langle TTTT \rangle, \langle J^a J^b J^c J^d \rangle, \langle TTJ^a J^b \rangle \]

Numerically ruled out

[Beem ML Liendo Rastelli van Rees]
What is the space of consistent SCFTs?

$4d$ $\mathcal{N} = 2$ SCFTs with $SU(2)$ flavor symmetry

$\langle TTTT \rangle$, $\langle J^a J^b J^c J^d \rangle$, $\langle TTJ^a J^b \rangle$

![Graph showing ruled out regions for $1/k_{4d}$ vs. $c_{4d}$](image)
Outline

Numerical conformal Bootstrap review

Chiral algebra

$4d$ $\mathcal{N} = 3$ SCFTs

Solving $\mathcal{N} = 3$ SCFTs?
4d $\mathcal{N} = 3$ SCFTs

→ *Non-trivial* interacting theories

[García-Etxebarria, Regalado] [Aharony, Tachikawa]
$4d \, \mathcal{N} = 3$ SCFTs

→ *Non-trivial* interacting theories

[García-Etxebarria, Regalado] [Aharony, Tachikawa]

→ Non-Lagrangian
$4d \mathcal{N} = 3$ SCFTs

→ Non-trivial interacting theories
  [García-Etxebarria, Regalado] [Aharony, Tachikawa]

→ Non-Lagrangian
  Properties from representation theory [Aharony, Evtikhiev]
$4d \ \mathcal{N} = 3$ SCFTs

→ Non-trivial interacting theories

[García-Etxebarria, Regalado] [Aharony, Tachikawa]

→ Non-Lagrangian

Properties from representation theory [Aharony, Evtikhiev]

→ $SU(3)_R \times U(1)_r$
4d $\mathcal{N} = 3$ SCFTs

$\rightarrow$ Non-trivial interacting theories

[García-Etxebarria, Regalado] [Aharony, Tachikawa]

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Properties from representation theory [Aharony, Evtikhiev]

$\rightarrow$ $SU(3)_R \times U(1)_r$

$\rightarrow$ No flavor symmetry
$4d \, \mathcal{N} = 3$ SCFTs

→ Non-trivial interacting theories
   [García-Etxebarria, Regalado] [Aharony, Tachikawa]

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→ $SU(3)_R \times U(1)_r$

→ No flavor symmetry

→ $c = a$
4d $\mathcal{N} = 3$ SCFTs

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  [García-Etxebarria, Regalado] [Aharony, Tachikawa]

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  Properties from representation theory [Aharony, Evtikhiev]

→ $SU(3)_R \times U(1)_r$

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→ No exactly marginal deformations
4d $\mathcal{N} = 3$ SCFTs

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  [García-Etxebarria, Regalado] [Aharony, Tachikawa]
→ Non-Lagrangian
  Properties from representation theory [Aharony, Evtikhiev]
→ $SU(3)_R \times U(1)_r$
→ No flavor symmetry
→ $c = a$
→ No exactly marginal deformations
→ Just another SCFT
Non-trivial interacting theories

[García-Etxebarria, Regalado] [Aharony, Tachikawa]

Non-Lagrangian
Properties from representation theory [Aharony, Evtikhiev]

$SU(3)_R \times U(1)_r$

No flavor symmetry

$c = a$

No exactly marginal deformations

Just another SCFT

as $\mathcal{N} = 2$: $SU(2)_R \times U(1)_r \times U(1)_F$
Outline

Numerical conformal Bootstrap review
Chiral algebra
$4d \mathcal{N} = 3$ SCFTs
Solving $\mathcal{N} = 3$ SCFTs?
Assume set of generators

→ Set of $\frac{1}{2}$-BPS operators
Chiral algebras for $\mathcal{N} = 3$ SCFTs

Assume set of generators

$\rightarrow$ Set of $\frac{1}{2}$-BPS operators
$\rightarrow$ Stress tensor supermultiplet

$\triangleright$ Can one write a consistent operator product algebra?
Chiral algebras for $\mathcal{N} = 3$ SCFTs

Assume set of generators

→ Set of $\frac{1}{2}$-BPS operators
→ Stress tensor supermultiplet

▶ Can one write a consistent operator product algebra?

▶ Simplest known theory:
Assume set of generators

- Set of $\frac{1}{2}$-BPS operators
- Stress tensor supermultiplet

Can one write a consistent operator product algebra?

Simplest known theory:

- $\frac{1}{2}$-BPS operators of dimension three
- Stress tensor
Assume set of generators

→ Set of $\frac{1}{2}$-BPS operators
→ Stress tensor supermultiplet

▷ Can one write a consistent operator product algebra?

▷ Simplest known theory:
→ $\frac{1}{2}$-BPS operators of dimension three
→ Stress tensor
→ chiral algebra fully fixed and $c_{4d} = \frac{15}{12}$ [Nishinaka, Tachikawa]
Chiral algebras for $\mathcal{N} = 3$ SCFTs

Assume set of generators

→ Set of $\frac{1}{2}$-BPS operators
→ Stress tensor supermultiplet

▷ Can one write a consistent operator product algebra?

▷ Simplest known theory:
  → $\frac{1}{2}$-BPS operators of dimension three
  → Stress tensor
  → chiral algebra fully fixed and $c_{4d} = \frac{15}{12}$ [Nishinaka, Tachikawa]

→ can compute additional protected OPE coefficients
Chiral algebras for $\mathcal{N} = 3$ SCFTs

Assume set of generators

→ Set of $\frac{1}{2}$-BPS operators
Assume set of generators

→ Set of $\frac{1}{2}$-BPS operators
→ Stress tensor supermultiplet

▷ What about higher rank theories?
Chiral algebras for $\mathcal{N} = 3$ SCFTs

Assume set of generators

$\rightarrow$ Set of $\frac{1}{2}$-BPS operators
$\rightarrow$ Stress tensor supermultiplet

$\blacktriangleright$ What about higher rank theories?
$\rightarrow$ This would be a closed subalgebra, but $c_{4d}$ is different
Chiral algebras for $\mathcal{N} = 3$ SCFTs

Assume set of generators

- Set of $\frac{1}{2}$-BPS operators
- Stress tensor supermultiplet

- What about higher rank theories?
  - This would be a closed subalgebra, but $c_{4d}$ is different
  - Assumed set of generators is incomplete for higher rank

[ML, Liendo, Meneghelli, Mitev]
Chiral algebras for $\mathcal{N} = 3$ SCFTs

Assume set of generators

→ Set of $\frac{1}{2}$-BPS operators
→ Stress tensor supermultiplet

▶ What about higher rank theories?
→ This would be a closed subalgebra, but $c_{4d}$ is different
→ Assumed set of generators is incomplete for higher rank
→ Minimum modification: add a single extra generator
Chiral algebras for $\mathcal{N} = 3$ SCFTs

Assume set of generators

$\rightarrow$ Set of $\frac{1}{2}$-BPS operators
$\rightarrow$ Stress tensor supermultiplet

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$\rightarrow$ This would be a closed subalgebra, but $c_{4d}$ is different
$\rightarrow$ Assumed set of generators is incomplete for higher rank
$\rightarrow$ Minimum modification: add a single extra generator
$\rightarrow$ constructed chiral algebra valid for any $c_{4d}$

[ML, Liendo, Meneghelli, Mitev]
Assume set of generators

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[ML, Liendo, Meneghelli, Mitev ]

subalgebra of higher rank theories?
Can we “zoom in” to the $c_{4d} = \frac{15}{12}$ SCFT?
Can we “zoom in” to the $c_{4d} = \frac{15}{12}$ SCFT?

- $\frac{1}{2}$-BPS operator of dimension three
Can we “zoom in” to the $c_{4d} = \frac{15}{12}$ SCFT?

- $\frac{1}{2}$-BPS operator of dimension three
- Step 1: Fix protected contributions
Solving $\mathcal{N} = 3$ SCFTs?

Can we “zoom in” to the $c_{4d} = \frac{15}{12}$ SCFT?

- $\frac{1}{2}$-BPS operator of dimension three
- Step 1: Fix protected contributions
  \[
  \lambda_{2d}^2 \sim \lambda_{\mathcal{O}4d}^2 - \lambda_{\mathcal{O}'4d}^2
  \]
Can we “zoom in” to the \( c_{4d} = \frac{15}{12} \) SCFT?

- \( \frac{1}{2} \)-BPS operator of dimension three
- Step 1: Fix protected contributions
  
  \[
  \lambda^2_{2d} \leadsto \lambda^2_{O_{4d}} - \lambda^2_{O'_{4d}}
  \]

  Valid for any \( \mathcal{N} = 3 \) SCFT with this operator
Solving $\mathcal{N} = 3$ SCFTs?

Can we “zoom in” to the $c_{4d} = \frac{15}{12}$ SCFT?

- $\frac{1}{2}$-BPS operator of dimension three
- Step 1: Fix protected contributions
  \[ \lambda_{2d}^2 \leadsto \lambda_{O_{4d}}^2 - \lambda_{O'_{4d}}^2 \]
  Valid for any $\mathcal{N} = 3$ SCFT with this operator
- $\mathcal{N} = 4$ “sits in the way”
Can we “zoom in” to the $c_{4d} = \frac{15}{12}$ SCFT?

- $\frac{1}{2}$-BPS operator of dimension three
- Step 1: Fix protected contributions
  \[ \lambda_{2d}^2 \sim \lambda_{O_{4d}}^2 - \lambda_{O_{4d}'}^2 \]
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- $\mathcal{N} = 4$ “sits in the way”
- Input chiral algebra data of specific theory
Can we “zoom in” to the $c_{4d} = \frac{15}{12}$ SCFT?

- $\frac{1}{2}$-BPS operator of dimension three
- Step 1: Fix protected contributions
  \[ \lambda_{2d}^2 \sim \lambda_{\mathcal{O}_{4d}}^2 - \lambda_{\mathcal{O}_{4d}'}^2 \]
  Valid for any $\mathcal{N} = 3$ SCFT with this operator
- $\mathcal{N} = 4$ “sits in the way”
- Input chiral algebra data of specific theory
  chiral algebra conjectured by [Nishinaka, Tachikawa]
$4d \mathcal{N} = 3$ SCFT with $c = \frac{15}{12}$

\[ \ell = 0, \quad \Delta_{[1,1]} \geq 4 \]

Ruled out

$\mathcal{N} = 3$?

$\mathcal{N} = 4$ SYM

[ML, Liendo, Meneghelli, Mitev ]
$4d \mathcal{N} = 3 \text{ SCFTs}$

$\ell = 0, \Delta_{[1,1]} \geq 4$

Ruled out

$\mathcal{N} = 3$?

[ML, Liendo, Meneghelli, Mitev ]
$4d \quad \mathcal{N} = 3 \quad \text{SCFTs}$

\[ \ell = 0, \quad \Delta_{[1,1]} \geq 4 \]

Ruled out

$\mathcal{N} = 3$? No

[ML, Liendo, Meneghelli, Mitev]
$4d \mathcal{N} = 3$ SCFTs

$|\lambda_{B[3,3]}^\wedge|^2$

[ML, Liendo, Meneghelli, Mitev ]