Super p-Brane Theory

emerging from

Super Homotopy Theory

Urs Schreiber
(CAS Prague and HCM Bonn)

talk at String Math 2017

Based on	arXiv:1611.06536	with D. Fiorenza and H. Sati
	arXiv:1702.01774	with J. Huerta

these slides are kept at ncatlab.org/schreiber/print/StringMath2017

Notorious Open Problem of String Theory:

What is the full non-perturbative Theory?

Notorious Open Problem of String Theory:

What is the full non-perturbative Theory?

We still have no fundamental formulation of "M-theory" -
Work on formulating the fundamental principles underlying M-theory has noticeably waned. [. . .]. If history is a good guide, then we should expect that anything as profound and far-reaching as a fully satisfactory formulation of M-theory is surely going to lead to new and novel mathematics. Regrettably, it is a problem the community seems to have put aside - temporarily. But, ultimately,
Physical Mathematics must return to this grand issue.

G. Moore, *Physical Mathematics and the Future*, at Strings 2014

Notorious Open Problem of String Theory:

What is the full non-perturbative Theory?

What is even its **Principle**?

Principles

	physics	mathematics
	gauge principle	homotopy theory
&	Pauli exclusion	super-geometry
=		super-homotopy theory

Homotopy Theory

	stable		parameterized stable		plain
plain	spectra	$\hookrightarrow$ parameterized over the point $\hookrightarrow$	parameterized spectra	$\twoheadrightarrow$ underlying parameter space $\twoheadrightarrow$	spaces
rational					
super rational					

Homotopy Theory

	stable	parameterized stable	plain
plain	spectra ↪ (parameterized over the point)	parameterized spectra → (underlying parameter space)	spaces
rational			dgc-algebras
super rational			

Quillen 69, Sullivan 77 : infinitesimal methods in homotopy theory

Homotopy Theory

	stable		parameterized stable		plain
plain	spectra	$\hookrightarrow$ parameterized over the point	parameterized spectra	$\twoheadrightarrow$ underlying parameter space	spaces
rational					dgc-algebras
super rational					super dgc-algebras (“FDA”s)

Nieuwenhuizen 82, D’Auria-Fré 82: FDAs efficiently construct SuGra-s

Homotopy Theory

	stable		parameterized stable		plain
plain	spectra	parameterized over the point $\hookrightarrow$	parameterized spectra	underlying parameter space $\twoheadrightarrow$	spaces
rational	cochain complexes				dgc-algebras
super rational	super cochain complexes				super dgc-algebras (“FDA”s)

Schwede-Shipley 03 : stable homotopy theory subsumes homological algebra

Homotopy Theory

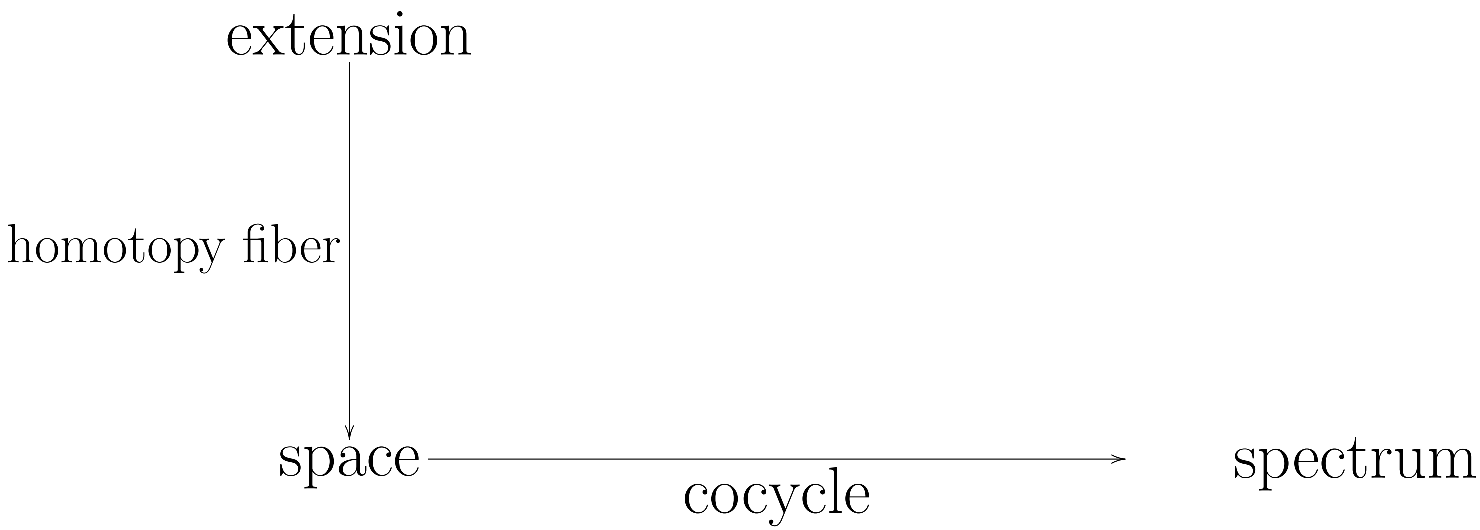
	stable	parameterized stable	plain
plain	spectra ↪ (parameterized over the point) →	parameterized spectra → (underlying parameter space) ↠	spaces
rational	cochain complexes	dg-modules	dgc-algebras
super rational	super cochain complexes	super dg-modules	super dgc-algebras (“FDA”s)

Schlegel

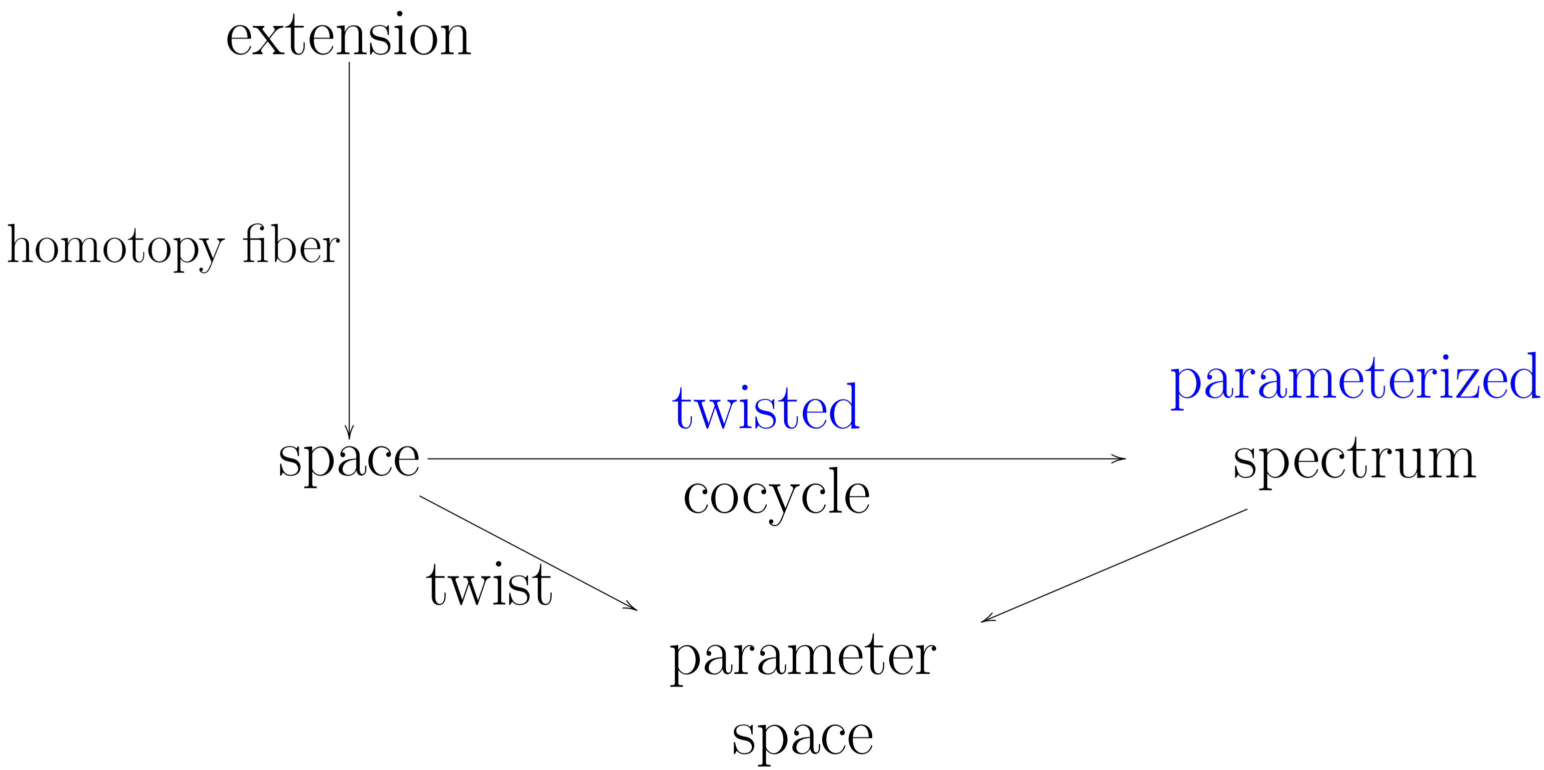
Homotopy Theory

	stable	parameterized stable	plain
plain	spectra ↪ (parameterized over the point)	parameterized spectra → (underlying parameter space)	spaces
rational	cochain complexes	dg-modules	dgc-algebras
super rational	super cochain complexes	super dg-modules	super dgc-algebras (“FDA”s)

Cohomology



Cohomology



Nikolaus-Schreiber-Stevenson 12, Ando-Blumberg-Gepner-Hopkins-Rezk 14

We now work out
in *rational* super-homotopy theory
a tower of extensions,
each invariant wrt
automorphisms modulo R-symmetries.

We now work out

in *rational* super-homotopy theory

a tower of extensions,

each invariant wrt

automorphisms modulo R-symmetries.

Beware:
Everything in the following holds
in (super-) *rational* homotopy theory.

In the beginning
the atom of space:

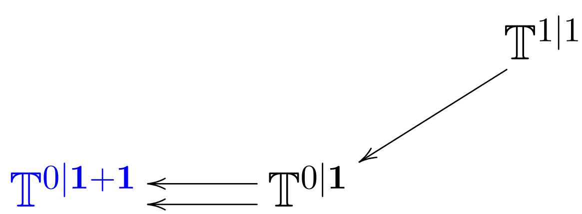
the superpoint

$\mathbb{T}^{0|\mathbf{1}}$

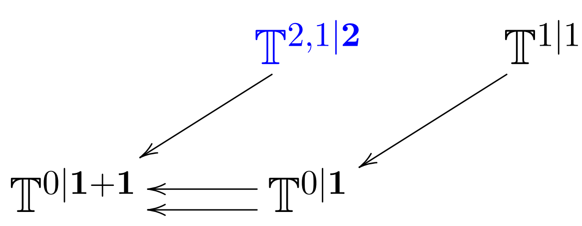
Its maximal torus extension
is the super-line

$$\mathbb{T}^{1|1} \longrightarrow \mathbb{T}^{0|\mathbf{1}}$$

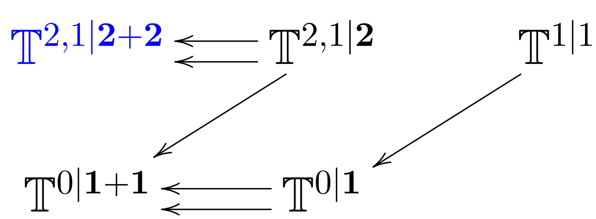
its type II version:
the $N = 2$ superpoint



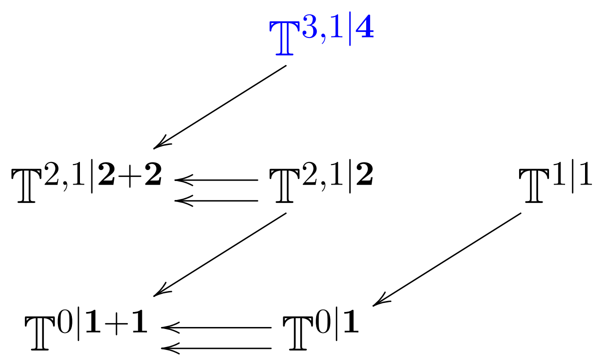
maximal torus extension:
$d = 3$, $N = 1$
super-Minkowski spacetime

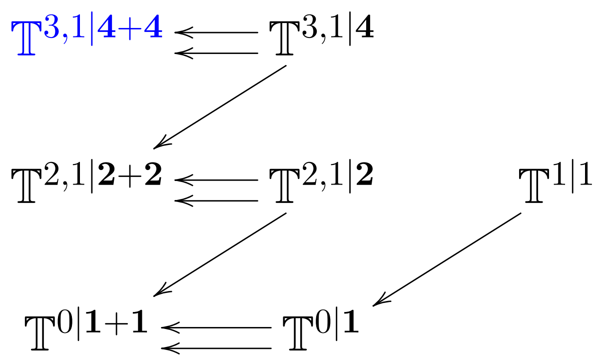


type II version:
$d = 3$, $N = 2$
super-Minkowski spacetime

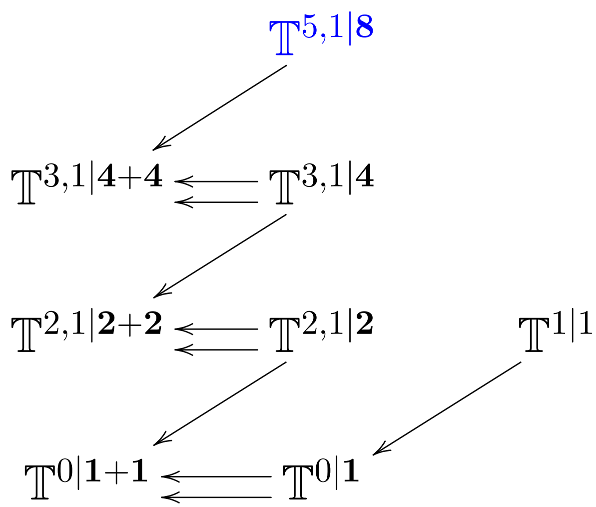


maximal invariant torus extension:
$d = 4$, $N = 1$
super-Minkowski spacetime

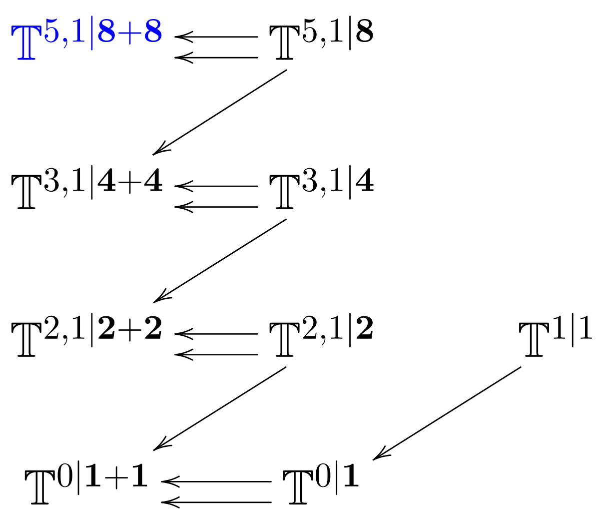




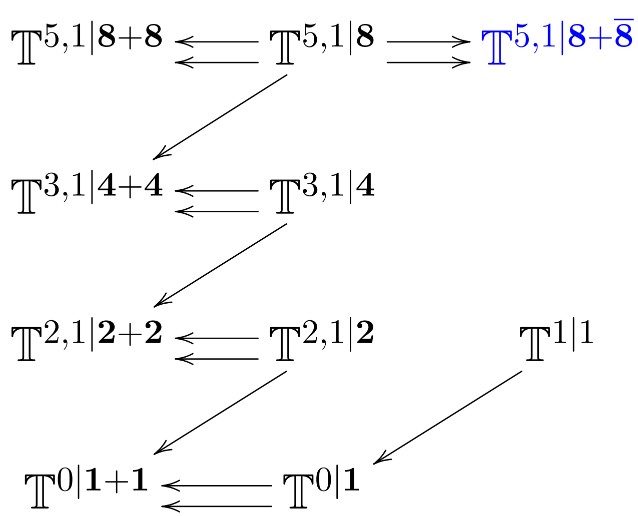
type II version:
$d = 4$, $N = 2$
super-Minkowski spacetime



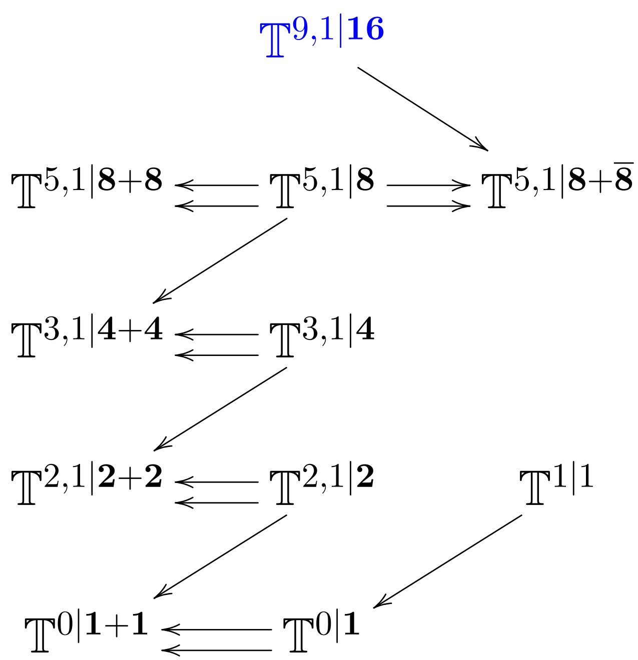
maximal invariant torus extension:
$d = 6$, $N = 1$
super-Minkowski spacetime



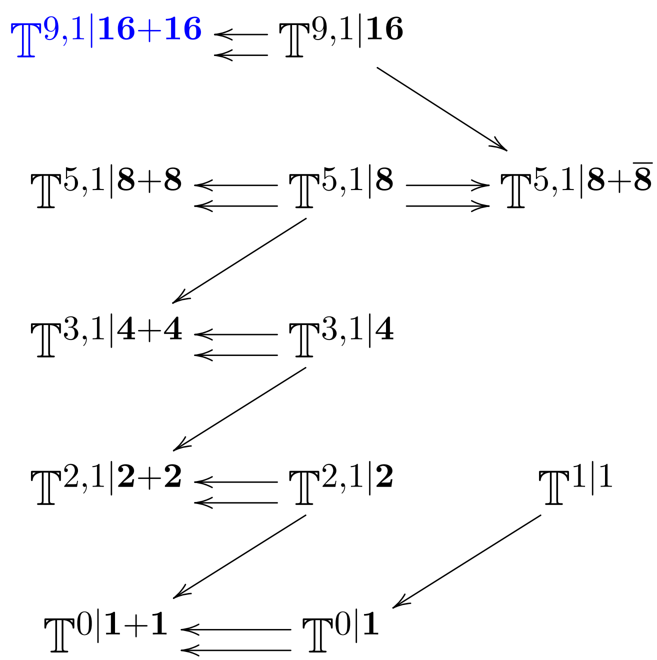
type IIB version:
$d = 6$, $N = (2, 0)$
super-Minkowski spacetime.



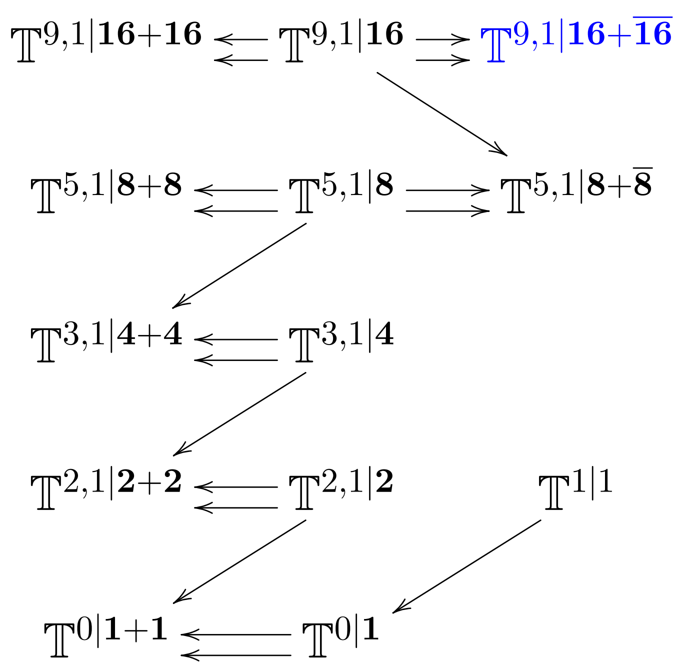
type IIA version:
$d = 6$, $N = (1, 1)$
super-Minkowski spacetime.



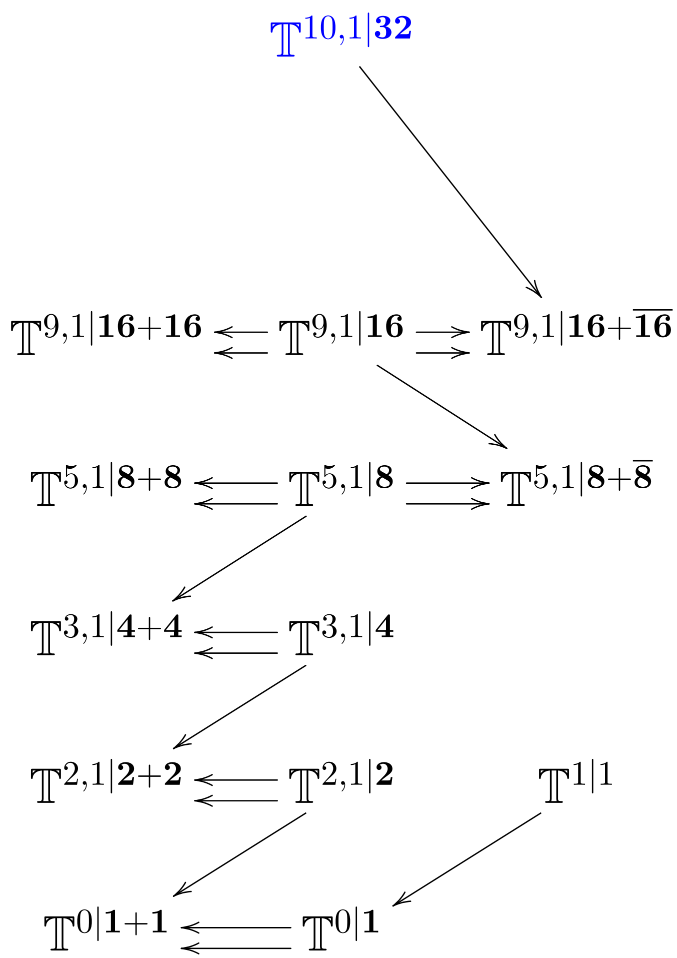
maximal invariant torus extension:
$d = 10$, $N = 1$
super-Minkowski spacetime



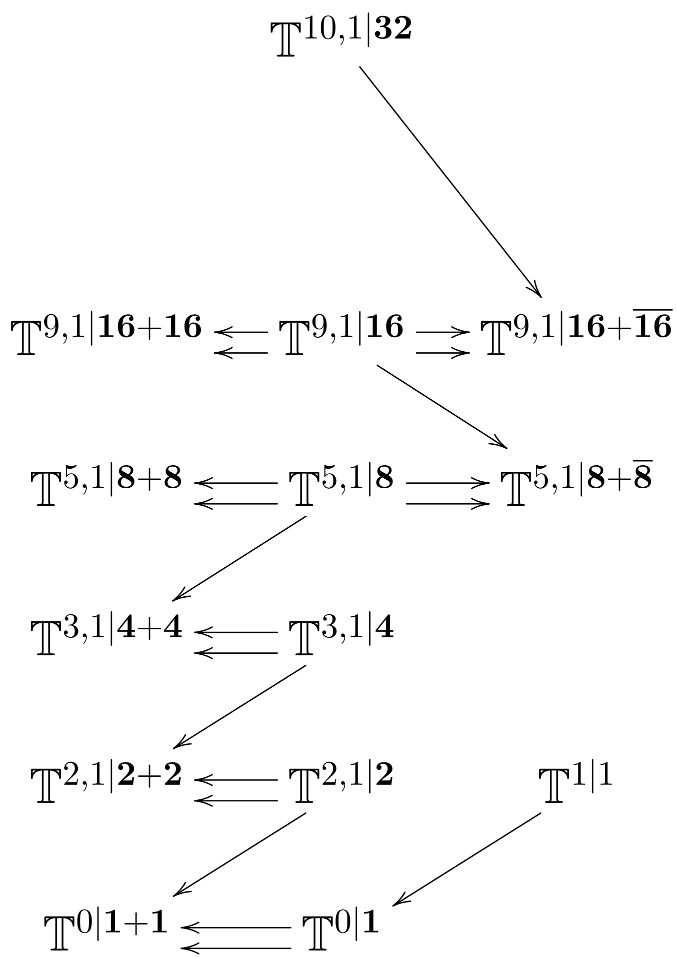
type IIB version:
$d = 10$, $N = (2, 0)$
super-Minkowski spacetime



and its type IIA version:
$d = 10$, $N = (1,1)$
super-Minkowski spacetime

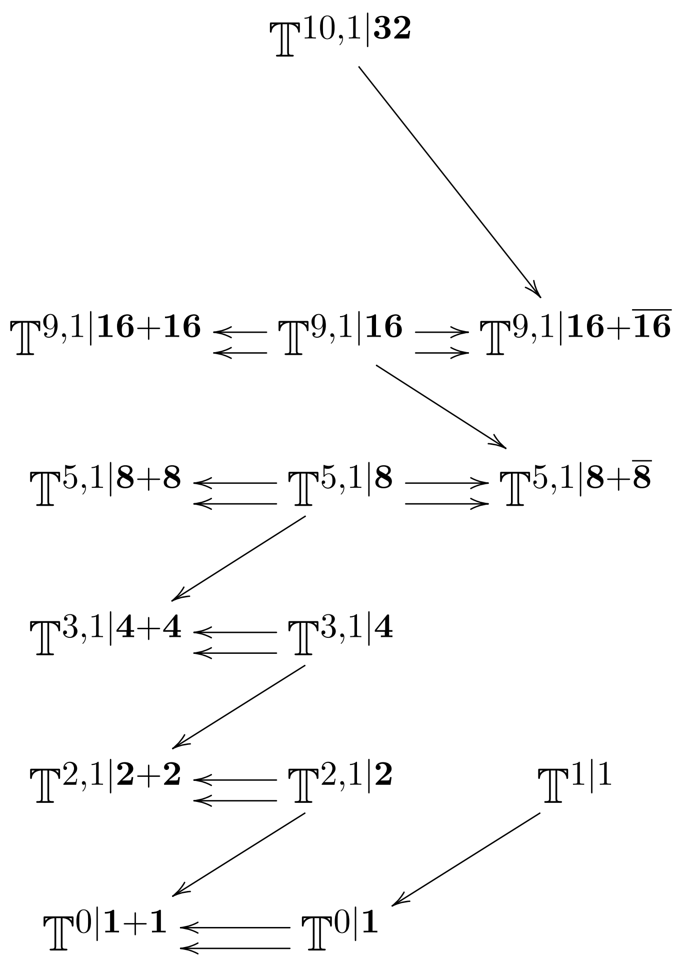


maximal invariant torus extension:
$d = 11$, $N = 1$
super-Minkowski spacetime

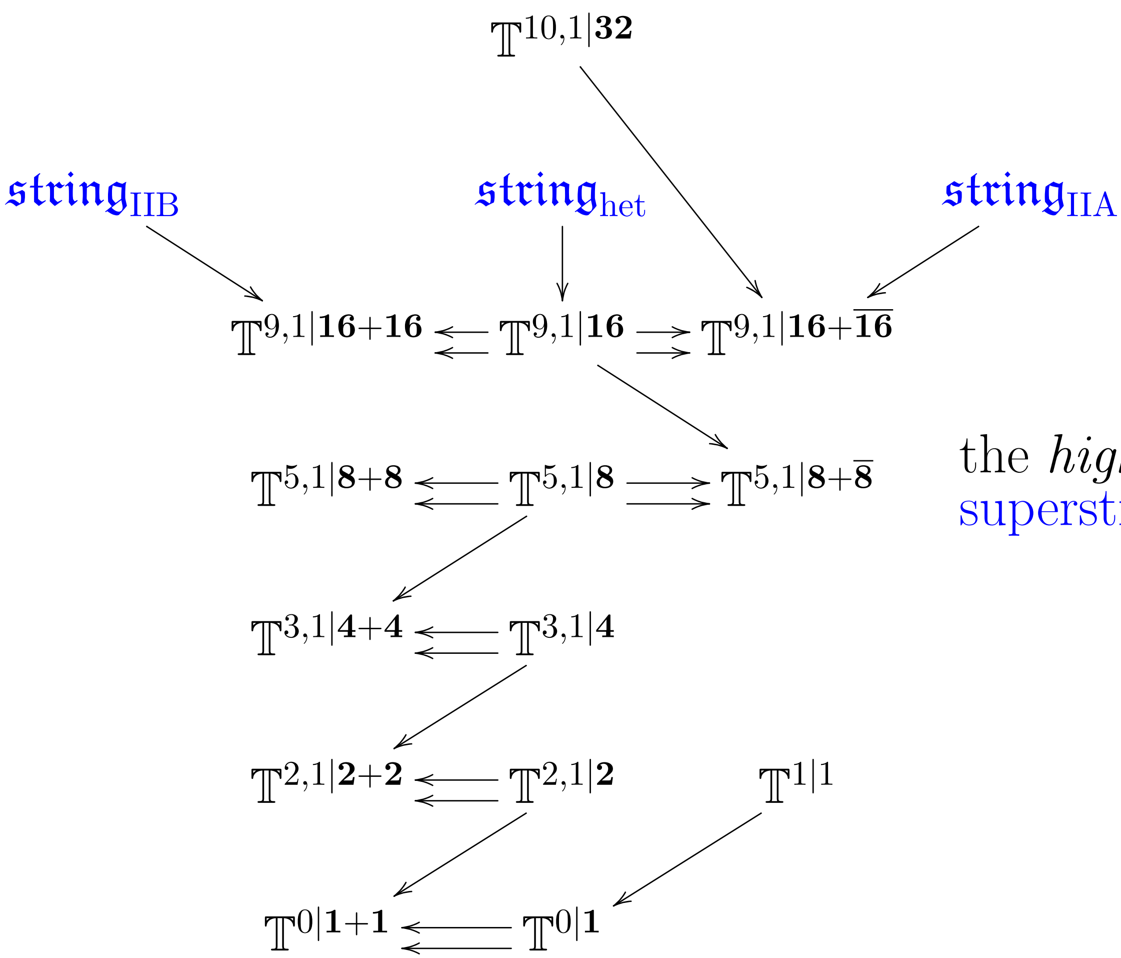


In summary:

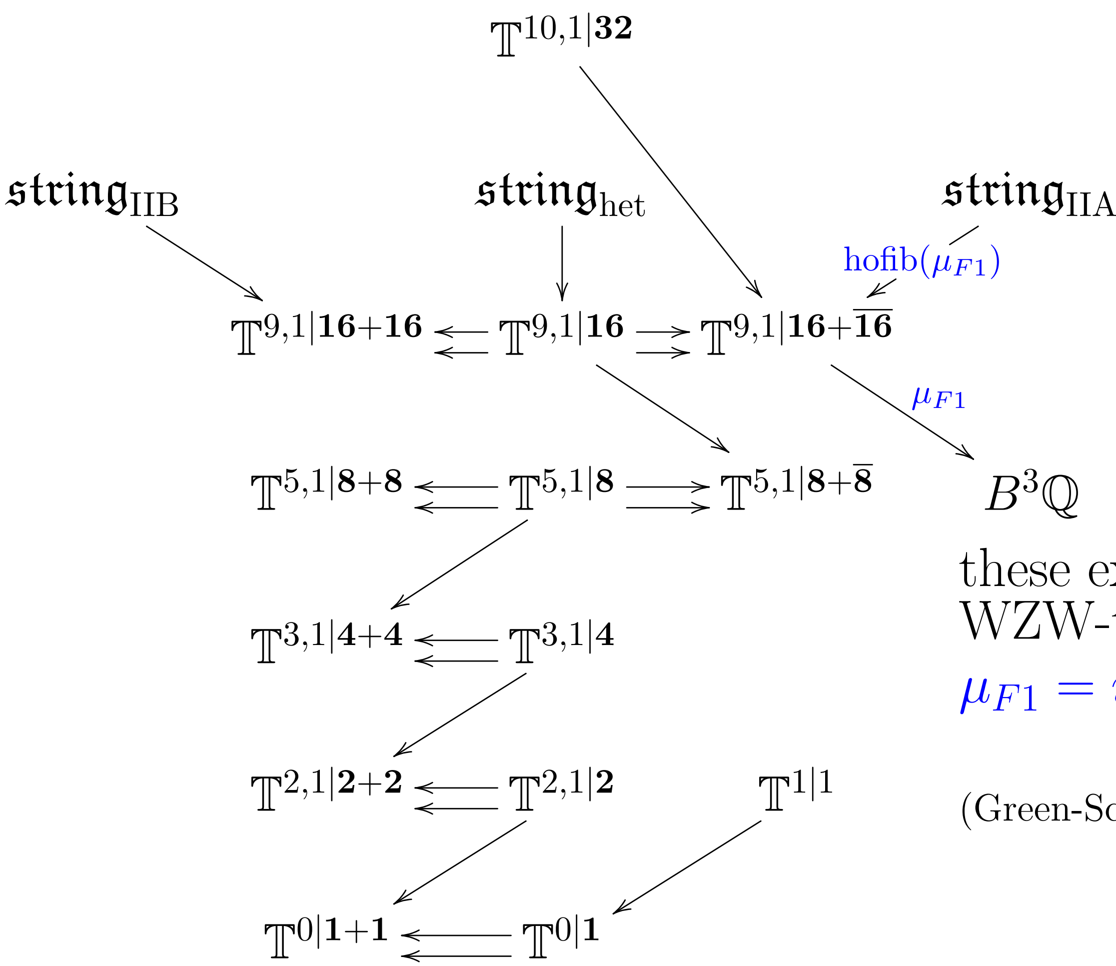
Theorem (Huerta-Schreiber 17):
There exists a diagram as shown
of maximal torus extensions
at each stage invariant
with respect to the semi-simple part
of automorphisms modulo R-symmetry
which happens to be
the Lorentzian Spin-groups.



$\mathbb{T}^{10,1|\mathbf{32}}$
$\mathbb{T}^{9,1|\mathbf{16}+\mathbf{16}}$
$\mathbb{T}^{9,1|\mathbf{16}}$
$\mathbb{T}^{9,1|\mathbf{16}+\overline{\mathbf{16}}}$
$\mathbb{T}^{5,1|\mathbf{8}+\mathbf{8}}$
$\mathbb{T}^{5,1|\mathbf{8}}$
$\mathbb{T}^{5,1|\mathbf{8}+\overline{\mathbf{8}}}$
$\mathbb{T}^{3,1|\mathbf{4}+\mathbf{4}}$
$\mathbb{T}^{3,1|\mathbf{4}}$
$\mathbb{T}^{2,1|\mathbf{2}+\mathbf{2}}$
$\mathbb{T}^{2,1|\mathbf{2}}$
$\mathbb{T}^{1|1}$
$\mathbb{T}^{0|\mathbf{1}+\mathbf{1}}$
$\mathbb{T}^{0|\mathbf{1}}$



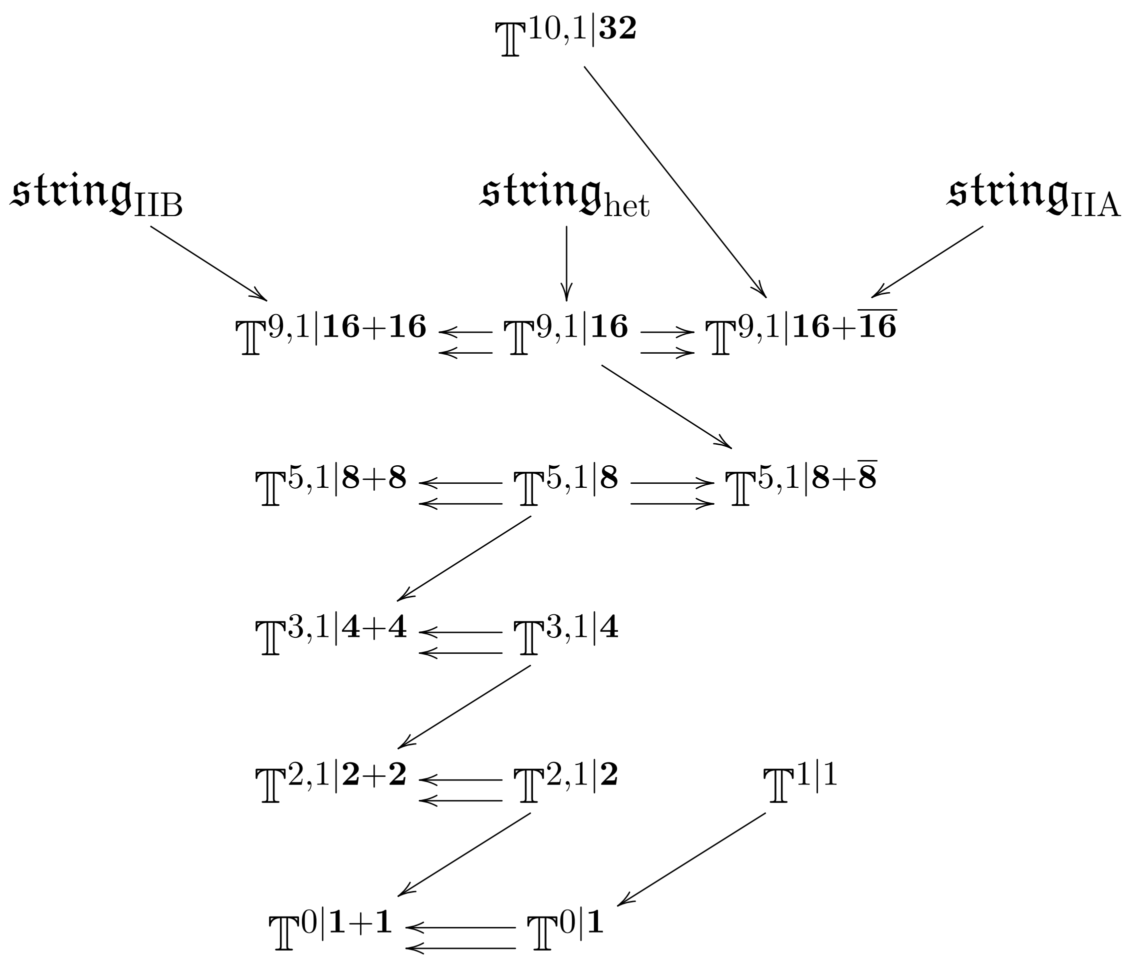
the *higher* invariant extensions:
superstrings condense



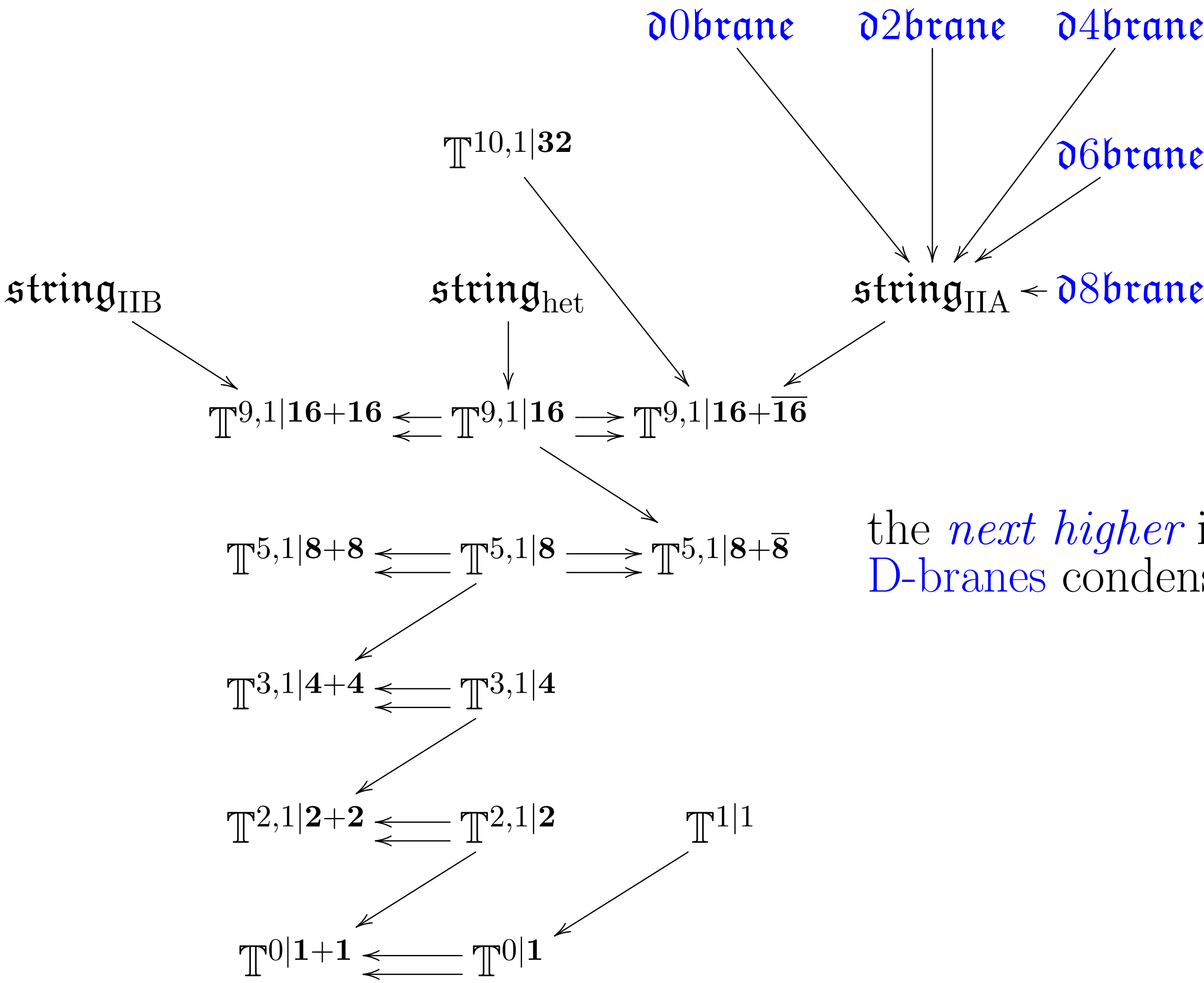
these extensions are classified by WZW-term for the GS-Superstring

$$\mu_{F1} = i\overline{\psi} \wedge \Gamma_a \psi \wedge e^a$$

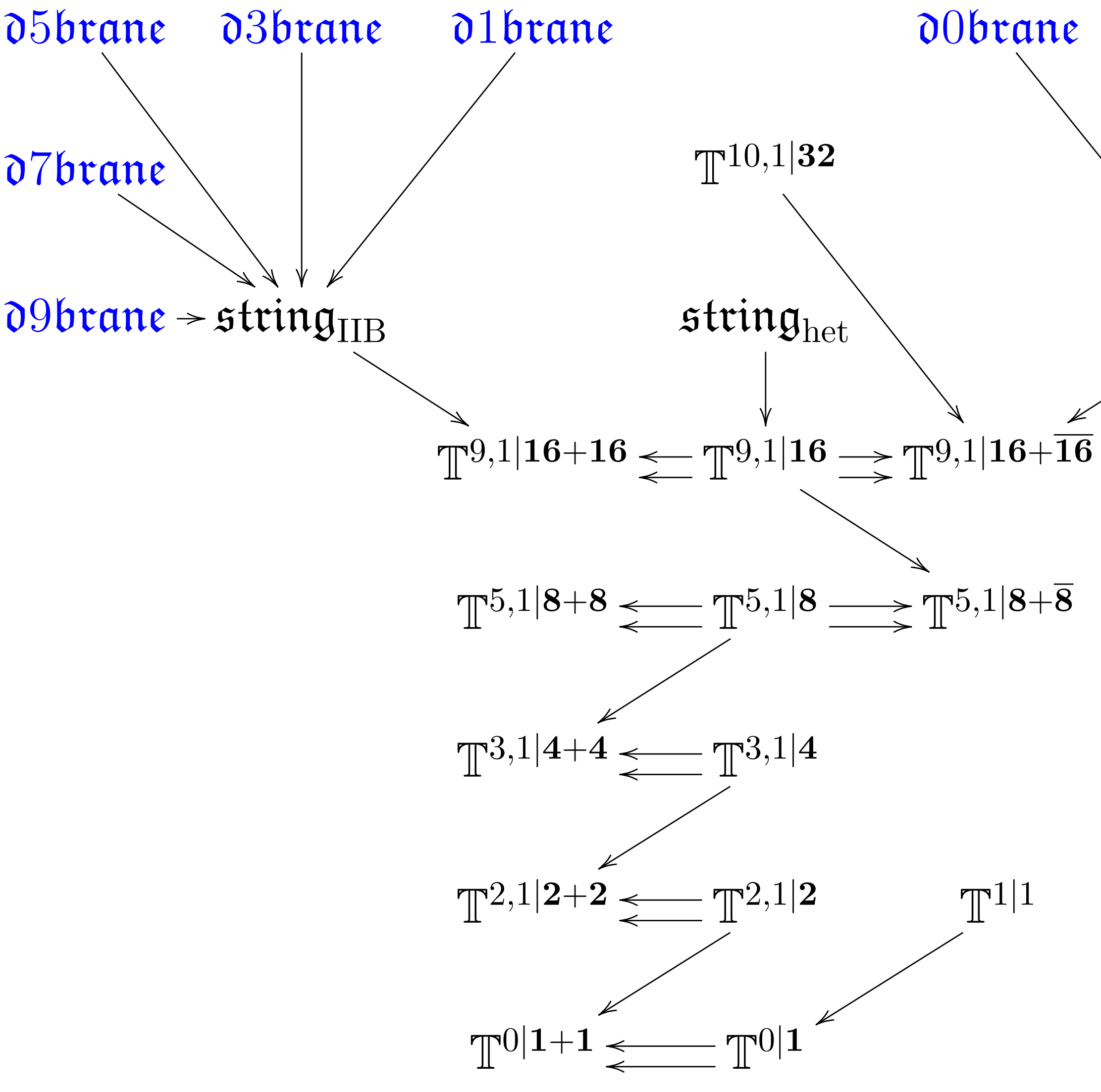
(Green-Schwarz 81, Henneaux-Mezincescu 85)



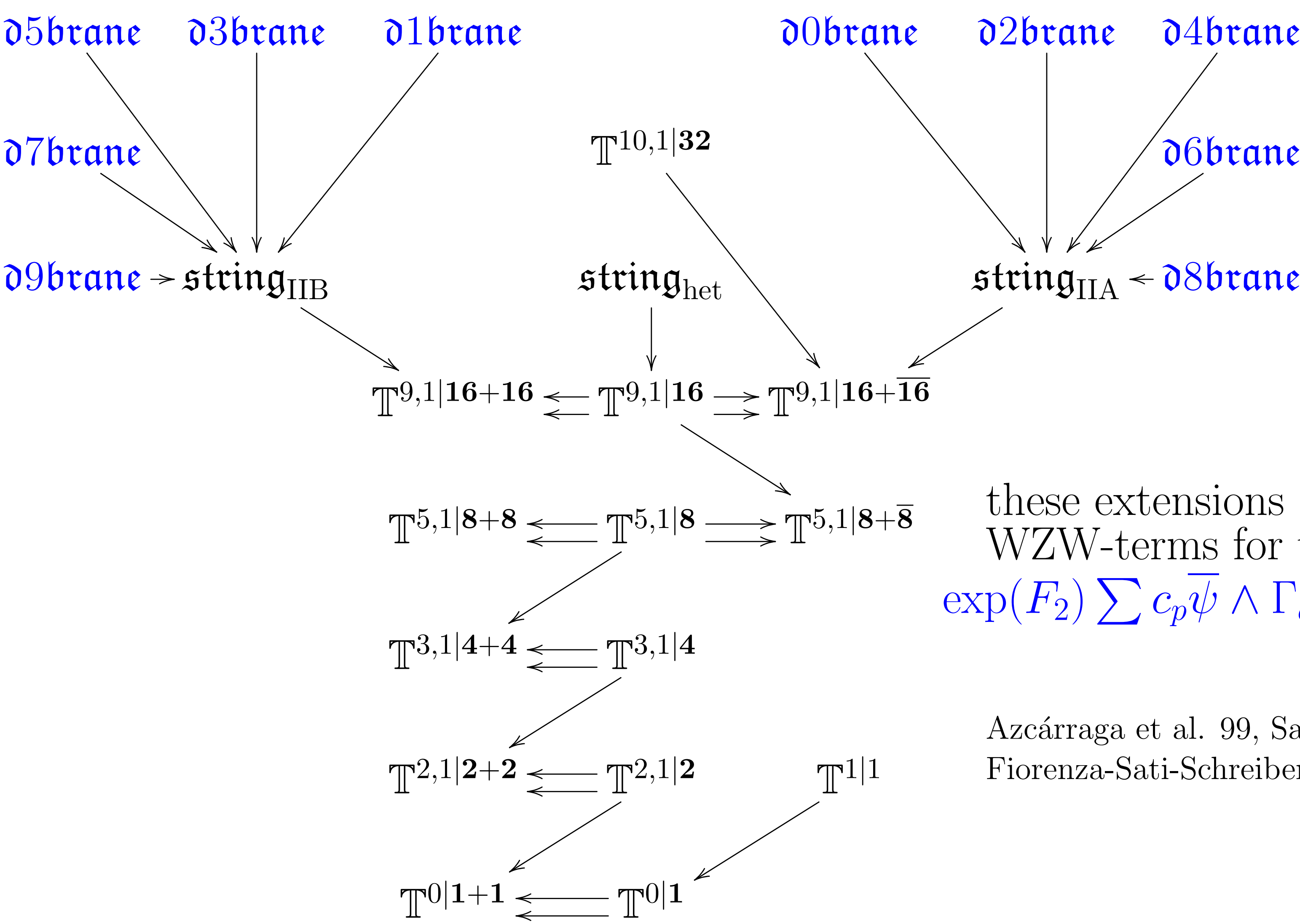
$\mathbb{T}^{10,1|\mathbf{32}}$
$\mathfrak{string}_{\mathrm{IIB}}$
$\mathfrak{string}_{\mathrm{het}}$
$\mathfrak{string}_{\mathrm{IIA}}$
$\mathbb{T}^{9,1|\mathbf{16}+\mathbf{16}}$
$\mathbb{T}^{9,1|\mathbf{16}}$
$\mathbb{T}^{9,1|\mathbf{16}+\overline{\mathbf{16}}}$
$\mathbb{T}^{5,1|\mathbf{8}+\mathbf{8}}$
$\mathbb{T}^{5,1|\mathbf{8}}$
$\mathbb{T}^{5,1|\mathbf{8}+\overline{\mathbf{8}}}$
$\mathbb{T}^{3,1|\mathbf{4}+\mathbf{4}}$
$\mathbb{T}^{3,1|\mathbf{4}}$
$\mathbb{T}^{2,1|\mathbf{2}+\mathbf{2}}$
$\mathbb{T}^{2,1|\mathbf{2}}$
$\mathbb{T}^{1|1}$
$\mathbb{T}^{0|\mathbf{1}+\mathbf{1}}$
$\mathbb{T}^{0|\mathbf{1}}$



the *next higher* invariant extensions:
D-branes condense



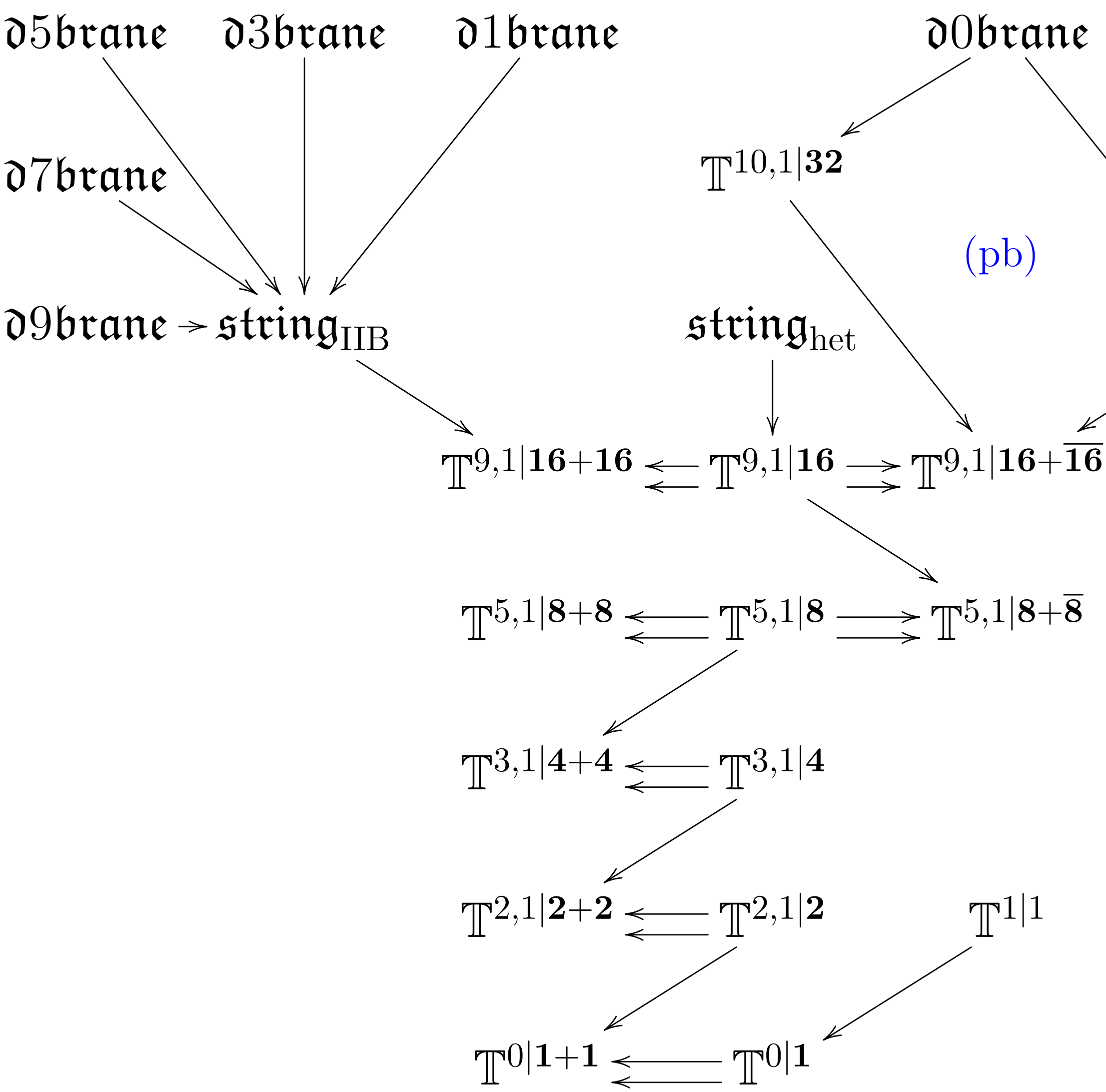
the *next higher* invariant extensions:
D-branes condense



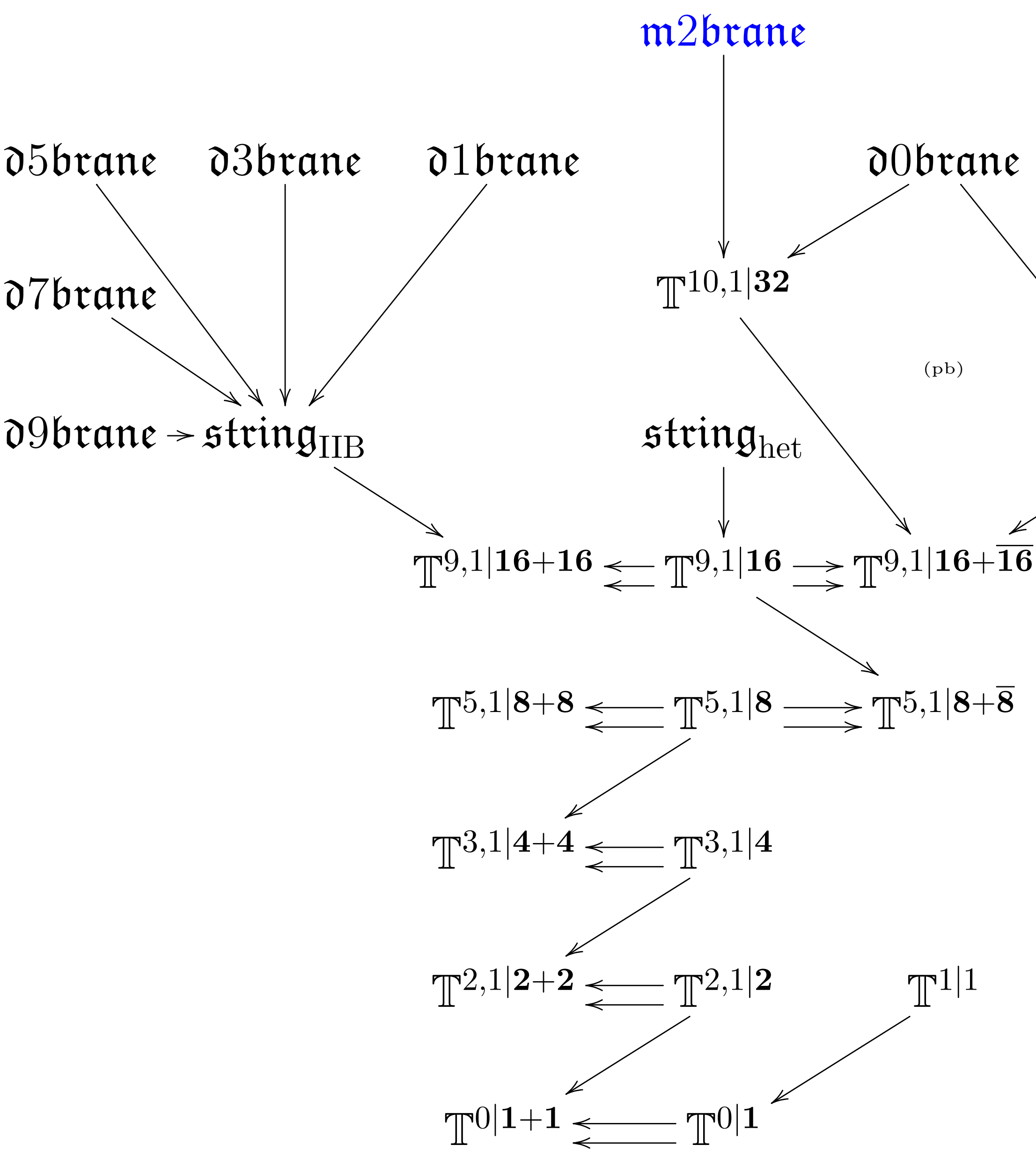
these extensions are classified by
WZW-terms for the super D-branes

$$\exp(F_2)\sum c_p \overline{\psi} \wedge \Gamma_{a_1\cdots a_p}\psi \wedge e_{a_1} \wedge \cdots \wedge e_{a_p}$$

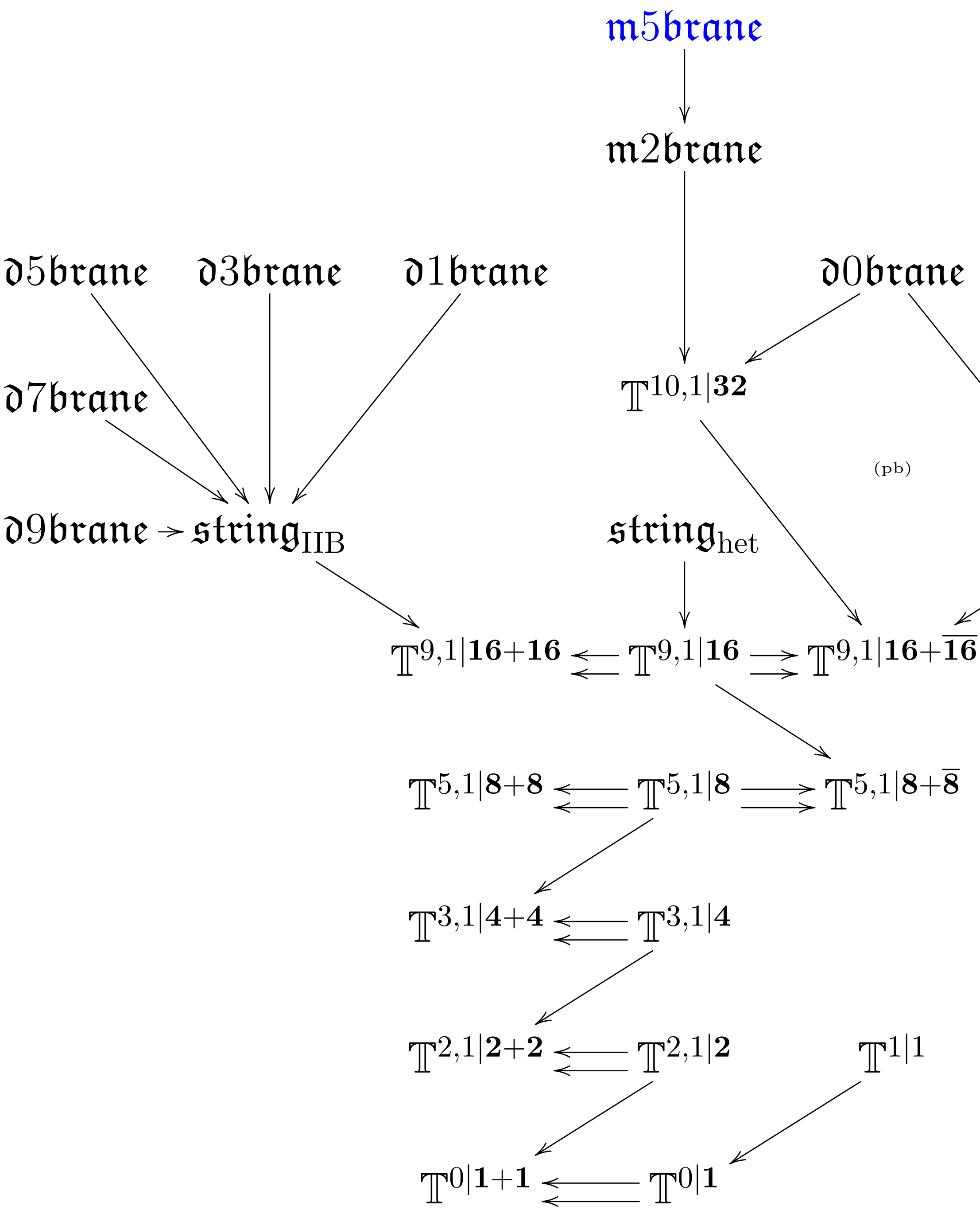
Azcárraga et al. 99, Sakaguchi 00
Fiorenza-Sati-Schreiber 13



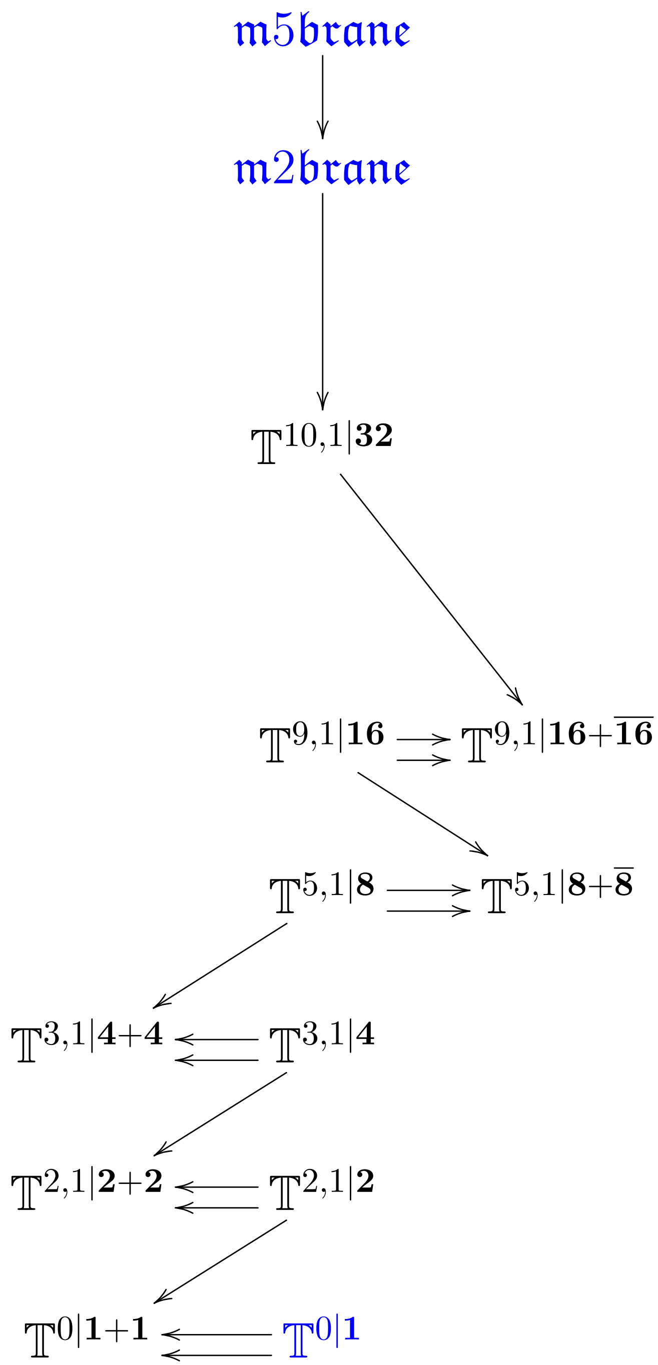
D0-brane condensate
is 11-th dimension
via homotopy pullback



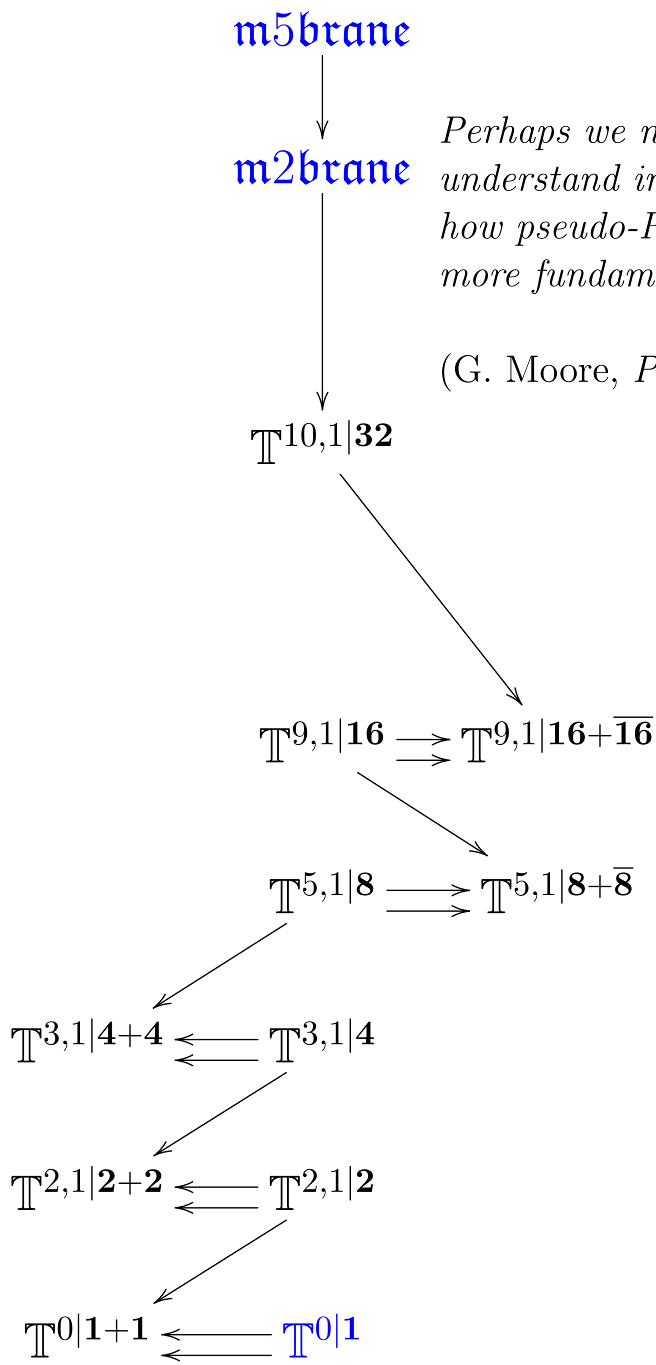
its own *higher* invariant extension:
M2-branes condense



its *next higher* extension:
M5-branes condense



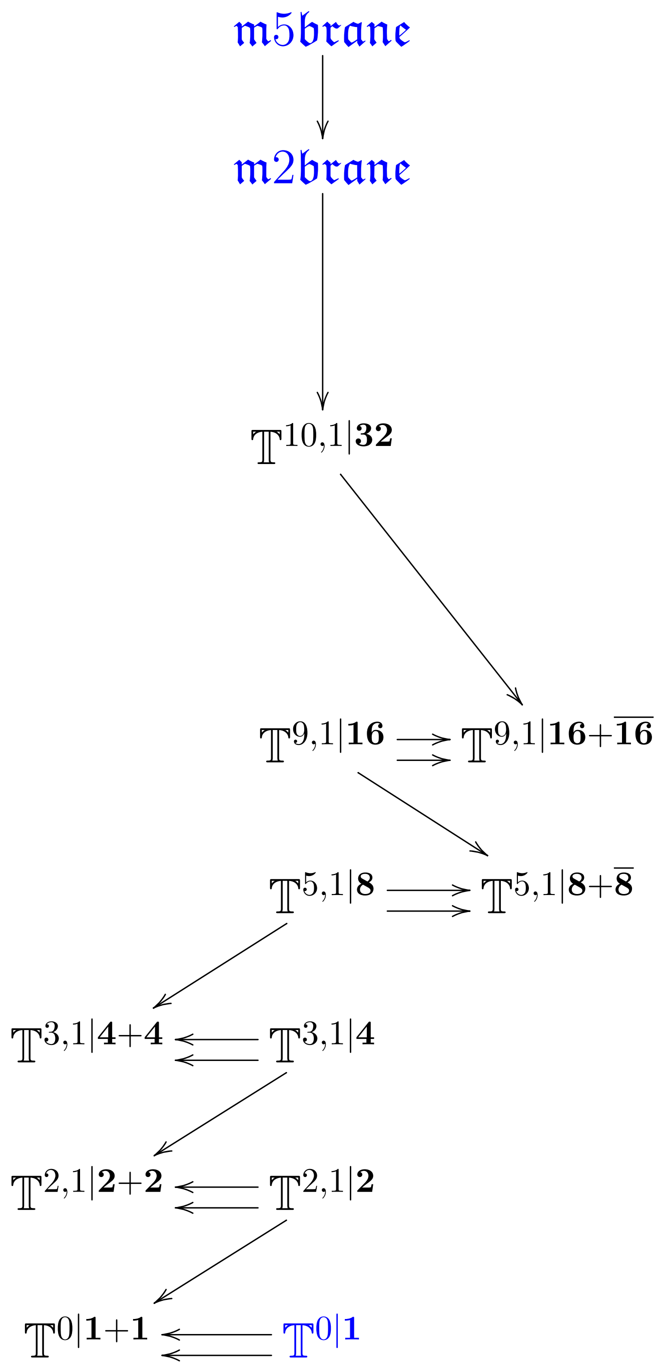
spacetime and M-branes have emerged from the superpoint as iterated higher invariant extensions



Perhaps we need to understand the nature of time itself better. [...]
understand in what sense time itself is an emergent concept, [...]
how pseudo-Riemannian geometry can emerge from
more fundamental and abstract notions such as categories of branes.

(G. Moore, *Physical Mathematics and the Future*, at Strings 2014)

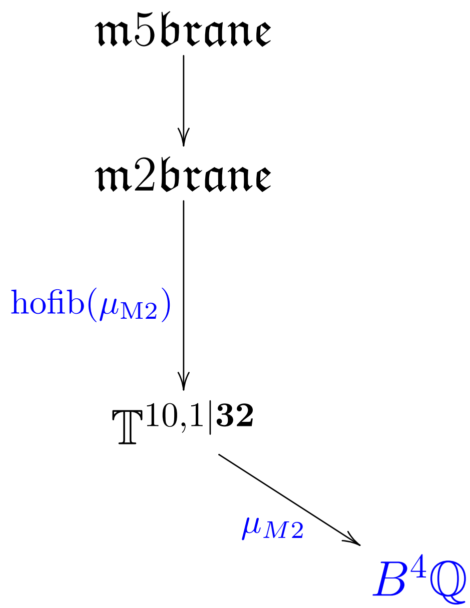
spacetime and M-branes
have emerged
from the superpoint
as iterated
higher invariant extensions



spacetime and M-branes
have emerged
from the superpoint
as iterated
higher invariant extensions

$$\begin{array}{c} \mathfrak{m5brane} \\ \downarrow \\ \mathfrak{m2brane} \\ \downarrow \\ \mathbb{T}^{10,1|\mathbf{32}} \end{array}$$

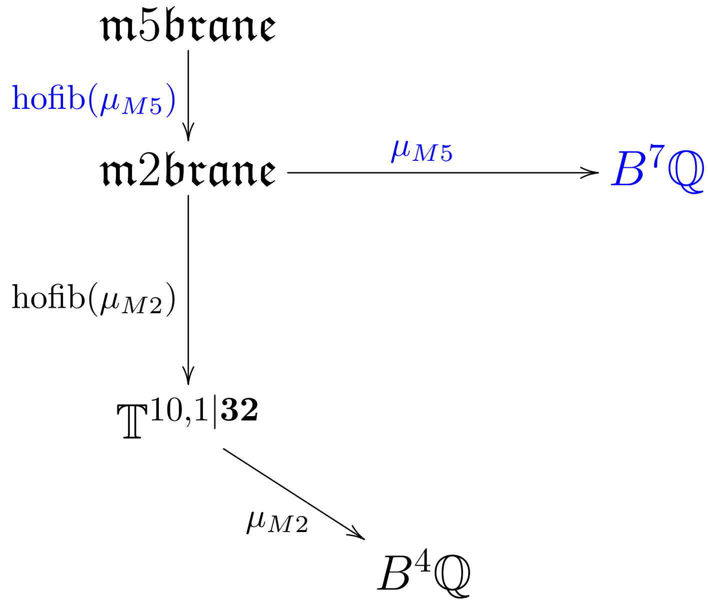
consider
the M-brane sector



the M2-extension is
classified by a 4-cocycle:
the GS-WZW-term of the M2-brane

$$\mu_{M2} = \tfrac{i}{2}\overline{\psi} \wedge \Gamma_{a_1 a_2}\psi \wedge e^{a_1} \wedge e^{a_2}$$

D'Auria-Fré 82 , Bergshoeff-Sezgin-Townsend 87,
Fiorenza-Sati-Schreiber 13

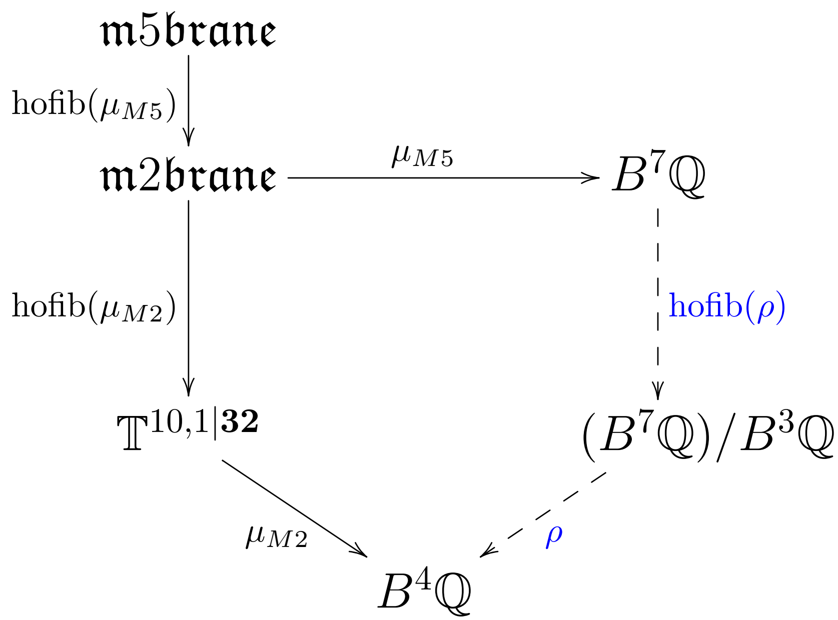


the M5-extension is
classified by a 7-cocycle:

the GS-WZW-terms of the M5-brane

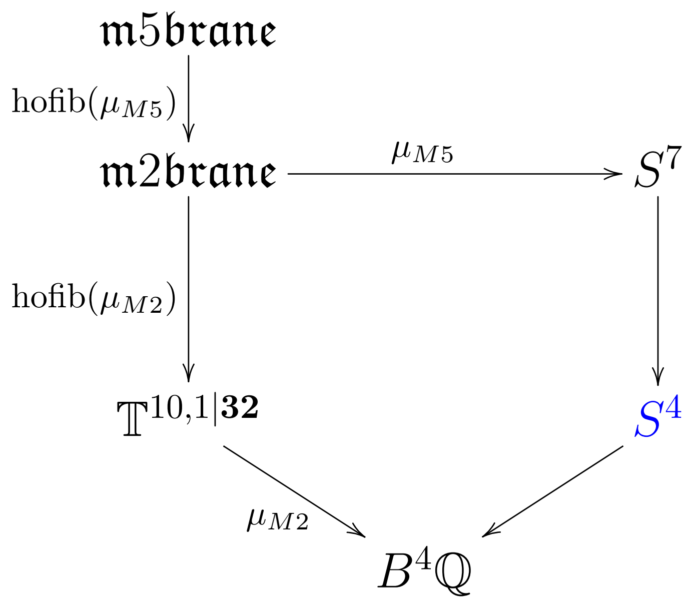
$$\mu_{M5} = \tfrac{1}{5!}\overline{\psi} \wedge \Gamma_{a_1 \cdots a_5}\psi \wedge e^{a_1} \wedge \cdots \wedge e^{a_5} + \tfrac{1}{2}c_3 \wedge \tfrac{1}{2}\overline{\psi} \wedge \Gamma_{a_1 a_2}\psi \wedge e^{a_1} \wedge e^{a_2}$$

D'Auria-Fré 82, Pasti-Sorokin-Tonin 97,
Fiorenza-Sati-Schreiber 13



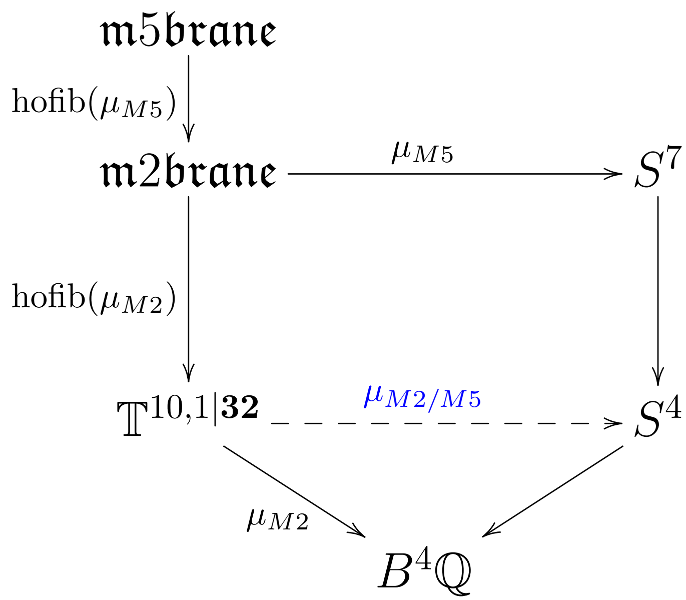
to descend this means
to ask for analogous fiber sequence
on the coefficients

Nikolaus-Schreiber-Stevenson 12



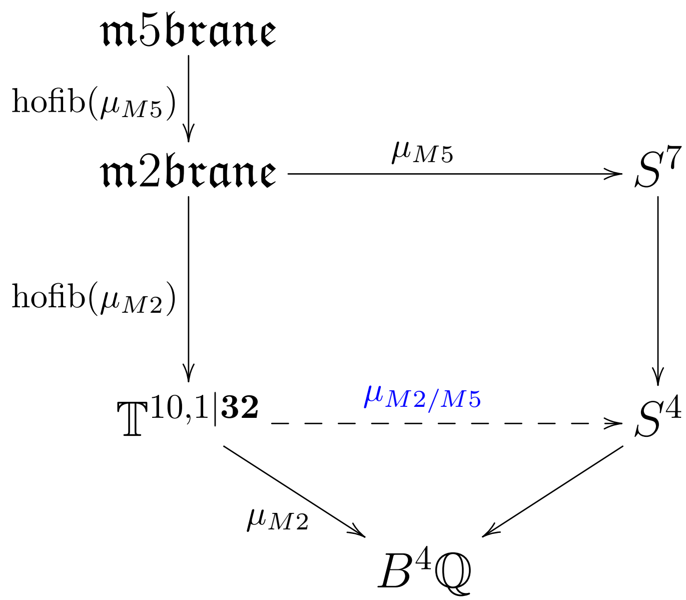
this comes out to be:
quaternionic Hopf fibration
(rationally)

Fiorenza-Sati-Schreiber 15



M5-cocycle descends:
unified M2/M5-cocycle

Fiorenza-Sati-Schreiber 15



dgc-model for S^4:
$$
\begin{aligned}
d\omega_4 &= 0 \\
d\omega_7 &= -\tfrac{1}{2}\omega_4 \wedge \omega_4
\end{aligned}
$$

11d SuGra C-field equation of motion: $\quad dG_7 + \frac{1}{2} G_4 \wedge G_4 = 0$

$$
\begin{array}{ccc}
\mathbb{T}^{10,1|\mathbf{32}} & \xrightarrow{\;\;\color{blue}{\mu_{M2/M5}}\;\;} & S^4 \\
& {\scriptstyle \mu_{M2}} \searrow \qquad \swarrow & \\
& B^4\mathbb{Q} &
\end{array}
$$

consider this

unified M-brane cocycle

$$
\begin{array}{ccccc}
\mathrm{Ext}\left(\mathbb{T}^{9,1|\mathbf{16}+\overline{\mathbf{16}}}\right) = \mathbb{T}^{10,1|\mathbf{32}} & & \xrightarrow{\quad \mu_{M2/M5} \quad} & & S^4 \\
& {\scriptstyle \mu_{M2}} \searrow & & \swarrow & \\
& & B^4\mathbb{Q} & &
\end{array}
$$

remember that

11d spacetime
is (maximal invariant) extension of
type IIA spacetime

$$
\begin{array}{ccccc}
\mathrm{Ext}\left(\mathbb{T}^{9,1\vert \mathbf{16}+\overline{\mathbf{16}}}\right) = \mathbb{T}^{10,1\vert\mathbf{32}} & \xrightarrow{\mu_{M2/M5}} & S^4 & = & \color{blue}{\mathrm{Ext}(S^4/S^1)} \\
{}_{\mu_{M2}}\searrow & & \swarrow & & \\
& B^4\mathbb{Q} & & &
\end{array}
$$

similarly S^4

is homotopy extension
of its S^1 homotopy quotient
via canonical $SU(2)$-action on
$S^4 \;\simeq\; S(\mathbb{R}\oplus\mathbb{H})$

$$
\begin{array}{ccccc}
\mathrm{Ext}\left(\mathbb{T}^{9,1|\mathbf{16}+\overline{\mathbf{16}}}\right) = \mathbb{T}^{10,1|\mathbf{32}} & \xrightarrow{\mu_{M2/M5}} & S^4 & = & \mathrm{Ext}(S^4/S^1) \\
& {\scriptstyle \mu_{M2}} \searrow & & \swarrow & \\
& & B^4\mathbb{Q} & &
\end{array}
$$

This orbifold $S^4/C_n \to S^4/S^1$
happens to be the same as
in the near-horizon geometry
of the black M5-brane
at an A-type singularity
Medeiros, Figueroa-O'Farrill 10

$$\begin{array}{ccccc} \mathrm{Ext}(\mathbb{T}^{9,1|\mathbf{16}+\overline{\mathbf{16}}}) & & \xrightarrow{\quad \mu_{M2/M5} \quad} & & \mathrm{Ext}(S^4/S^1) \\ & {\scriptstyle \mu_{M2}} \searrow & & \swarrow & \\ & & B^4\mathbb{Q} & & \end{array}$$

hence the

unified M2/M5-cocycle
is really of this form

$$
\begin{array}{ccc}
\mathrm{Ext}(\mathbb{T}^{9,1|\mathbf{16}+\overline{\mathbf{16}}}) & \xrightarrow{\quad \mu_{M2/M5} \quad} & \mathrm{Ext}(S^4/S^1) \\
 & {}_{\mu_{M2}}\searrow \qquad \swarrow & \\
 & B^4\mathbb{Q} &
\end{array}
$$

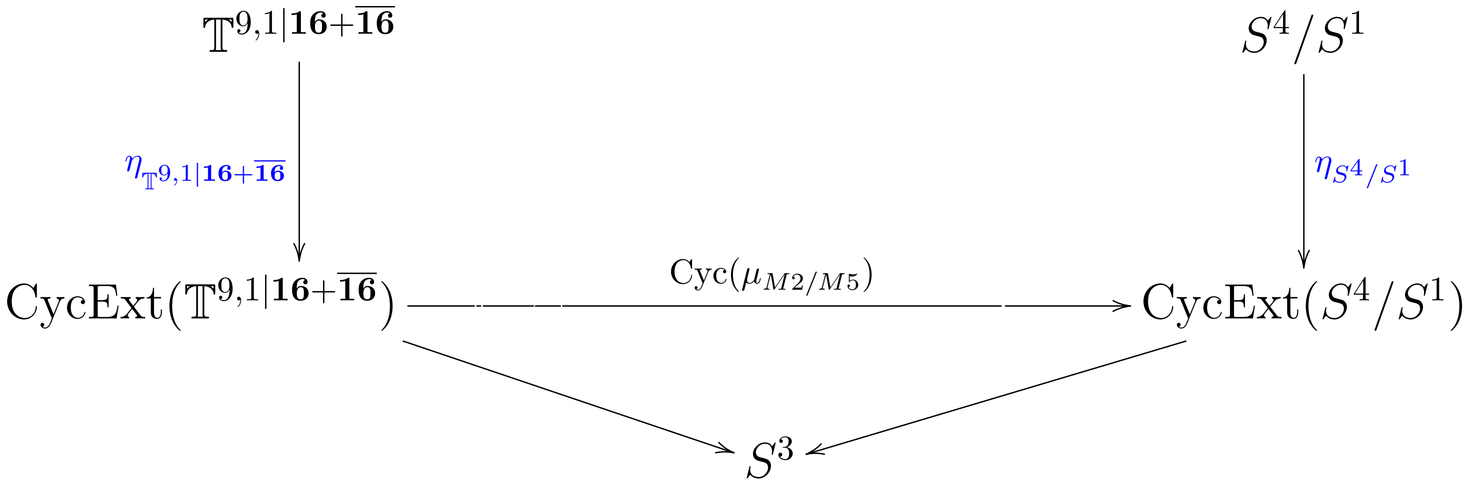
Theorem (Fiorenza-Sati-Schreiber 17): Ext has a derived right adjoint

$$
\mathrm{SuperHomotopyTypes} \underset{\mathrm{Cyclification}}{\overset{\mathrm{Extension}}{\underset{\longrightarrow}{\overset{\longleftarrow}{\bot}}}} \mathrm{SuperHomotopyTypes}_{/BS^1}
$$

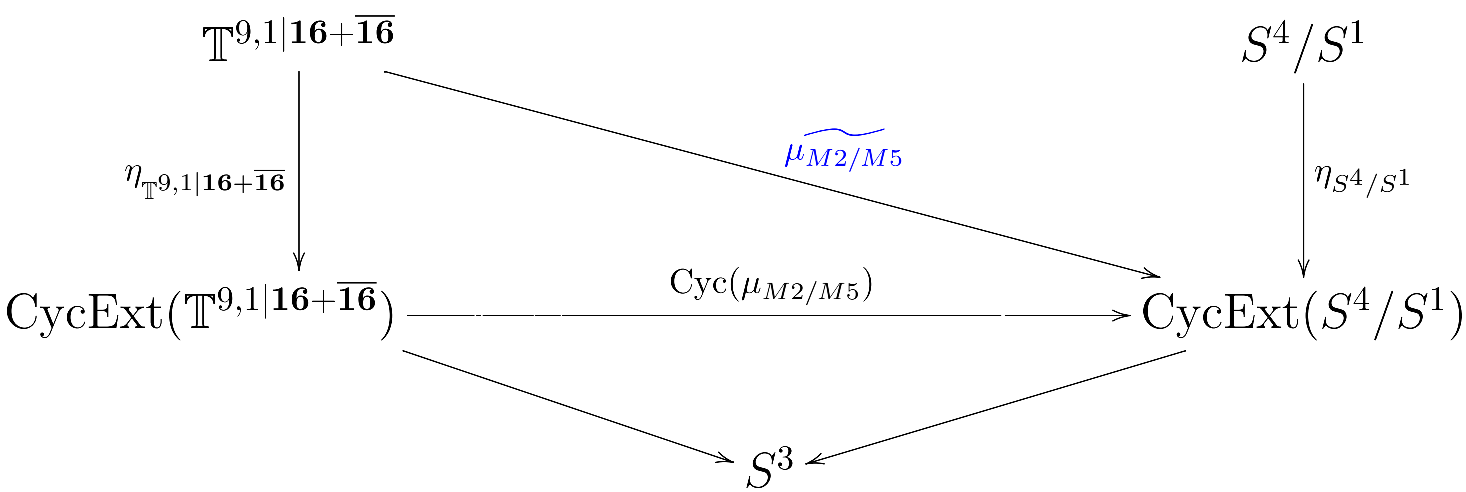
given by passing to twisted loop spaces / cyclic cohomology

$$
\begin{array}{ccc}
\mathrm{Cyc}\mathrm{Ext}\big(\mathbb{T}^{9,1|\mathbf{16}+\overline{\mathbf{16}}}\big) & \xrightarrow{\quad \mathrm{Cyc}(\mu_{M2/M5}) \quad} & \mathrm{Cyc}\mathrm{Ext}(S^4/S^1) \\
 & {\scriptstyle \mu_{F1}} \searrow \qquad \swarrow & \\
 & S^3 &
\end{array}
$$

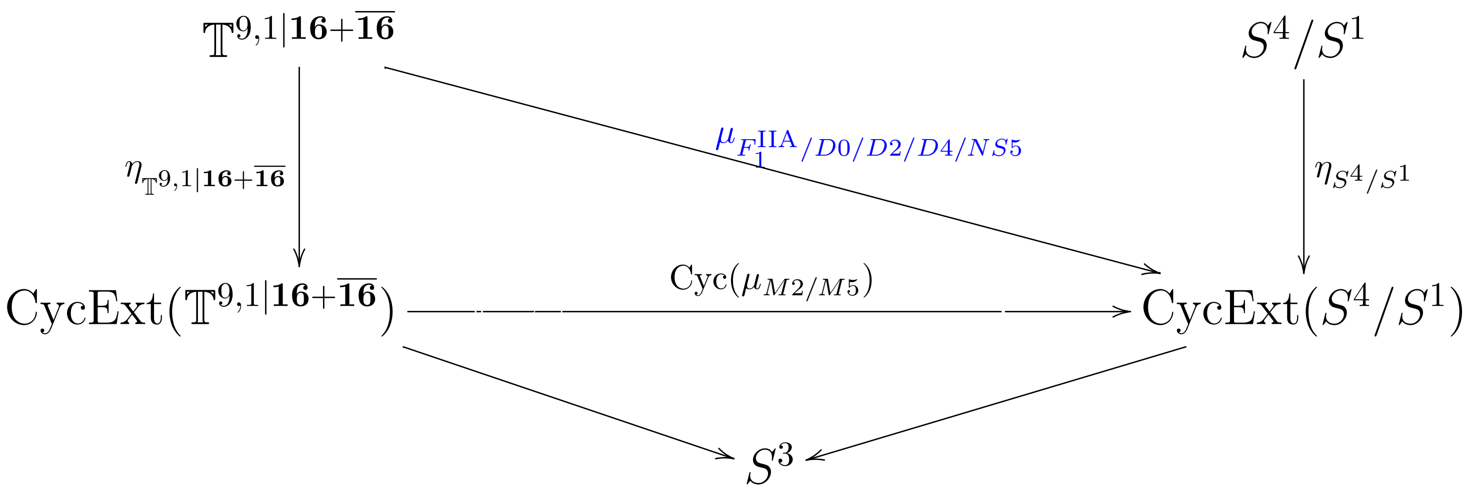
apply the right adjoint



and compose
with the adjunction unit

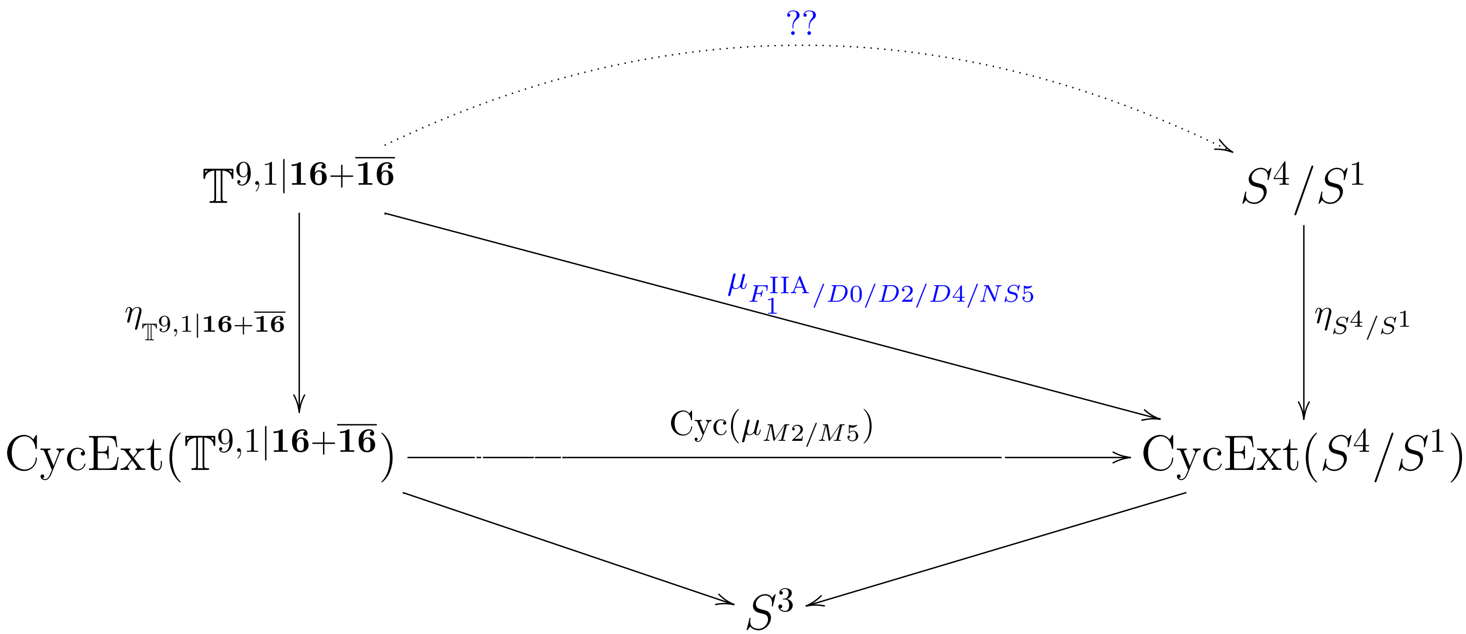


to obtain the
Ext $\dashv$ Cyc-adjunct
of the unified M-brane cocycle



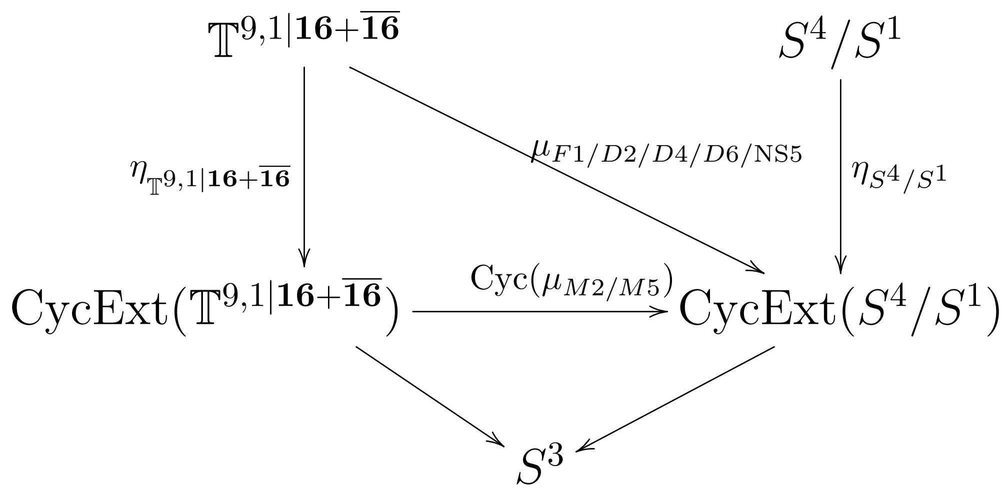
Theorem (Fiorenza-Sati-Schreiber 17) : This is the Green-Schwarz WZW term of the double dimensional reduction of M2/M5 to $F_1^{\mathrm{IIA}}/D0/D2/D4/\mathrm{NS5}$:

dgc-algebra for $\mathrm{CycExt}(S^4/S^1)$: $\left\{ \begin{array}{c} dH_3 = 0\ ,\ dH_7 = F_2 \wedge F_6 - \frac{1}{2} F_4 \wedge F_4 \\ dF_2 = 0\ ,\ dF_4 = H_3 \wedge F_2\ ,\ dF_6 = H_3 \wedge F_4 \end{array} \right.$

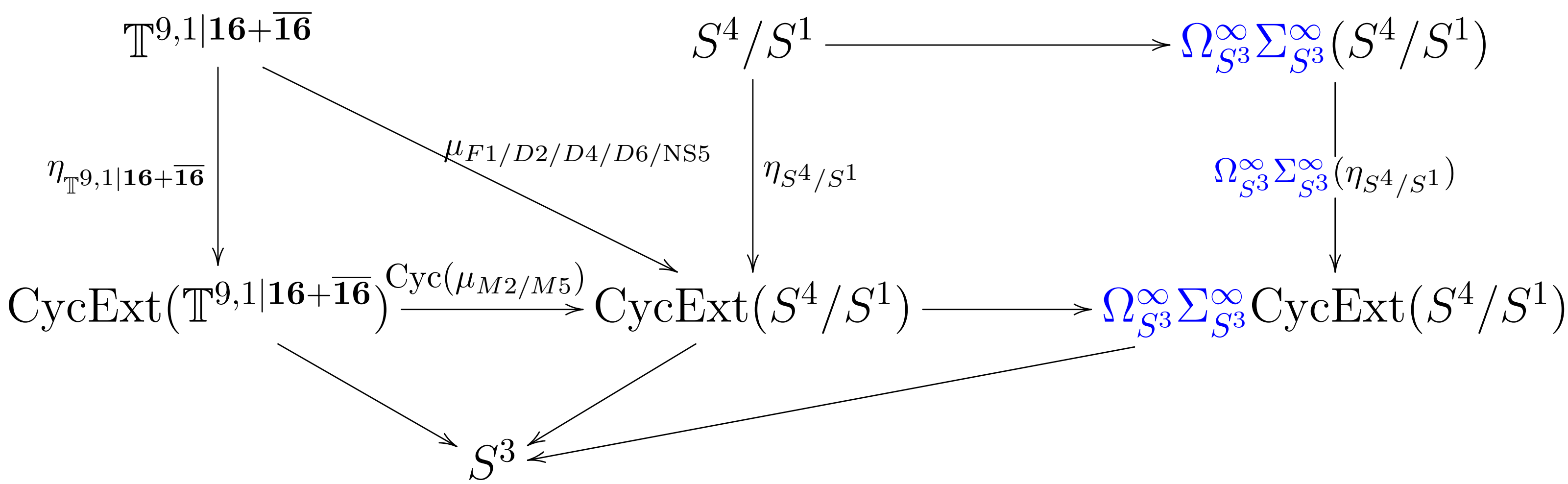


This gives rise to two questions:
1) Where are the $D(p \geq 6)$-branes (gauge enhancement)?

2) Is there a dashed lift as above?

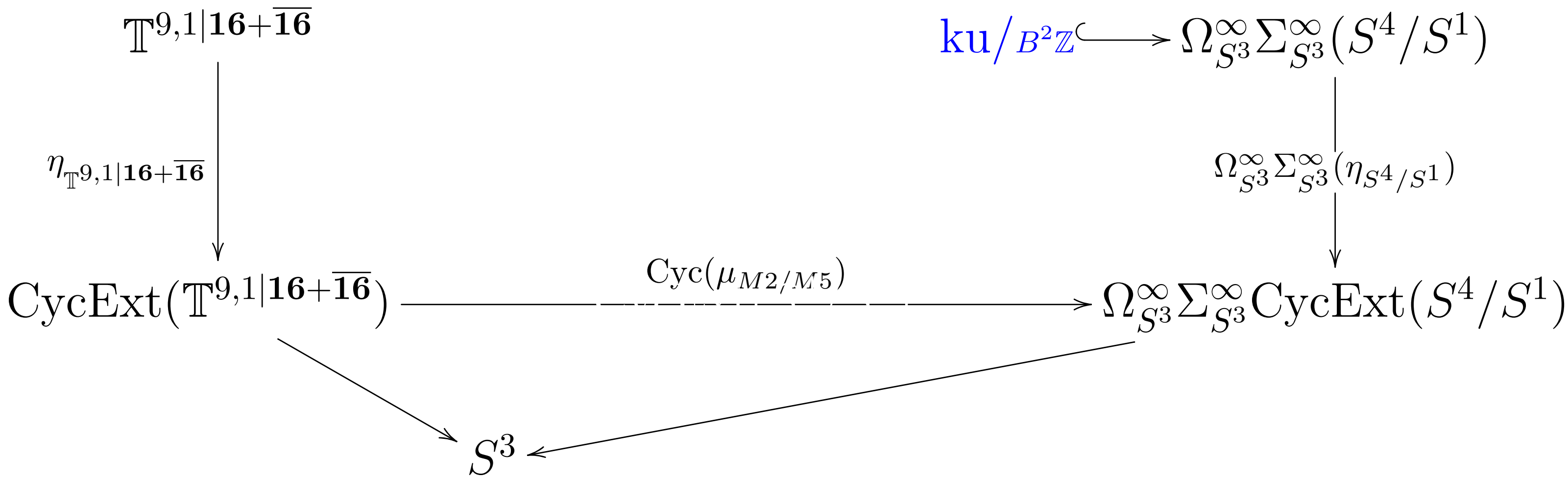


let us first make some room...



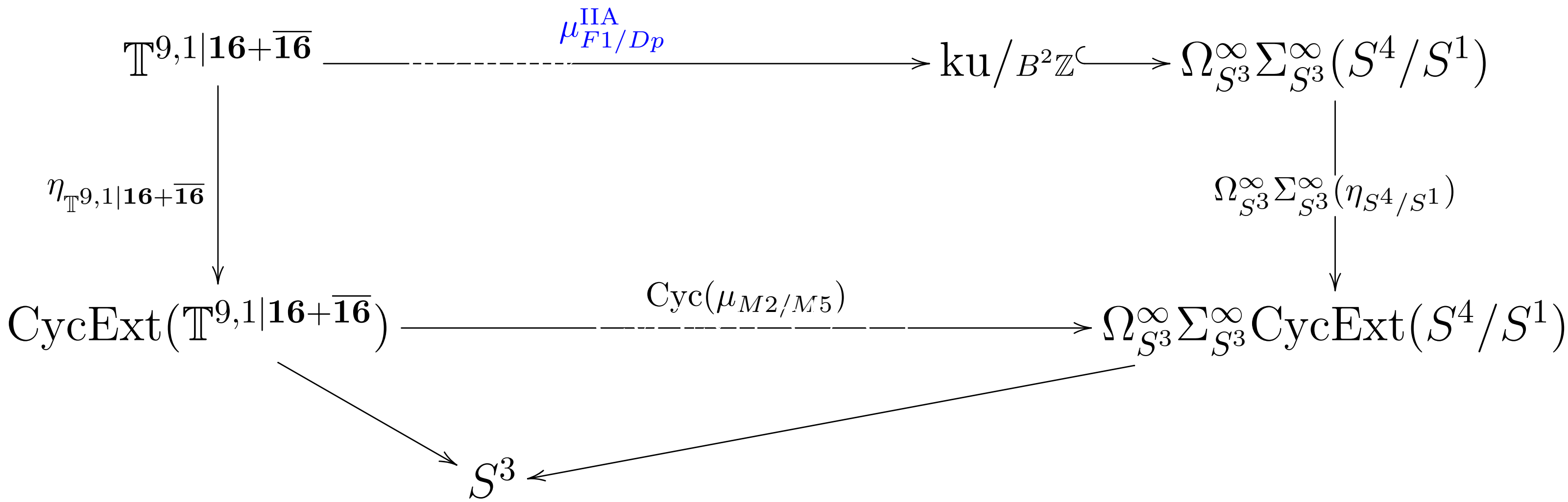
consider the Goodwillie-linearized lifting problem:
form the fiberwise suspension spectrum over S^3
to obtain an S^3 parameterized spectrum

$$
\begin{array}{ccc}
\mathbb{T}^{9,1|\mathbf{16}+\overline{\mathbf{16}}} & & \Omega^\infty_{S^3}\Sigma^\infty_{S^3}(S^4/S^1) \\
\Big\downarrow{\scriptstyle \eta_{\mathbb{T}^{9,1|\mathbf{16}+\overline{\mathbf{16}}}}} & & \Big\downarrow{\scriptstyle \Omega^\infty_{S^3}\Sigma^\infty_{S^3}(\eta_{S^4/S^1})} \\
\mathrm{CycExt}(\mathbb{T}^{9,1|\mathbf{16}+\overline{\mathbf{16}}}) & \xrightarrow{\ \mathrm{Cyc}(\mu_{M2/M5})\ } & \Omega^\infty_{S^3}\Sigma^\infty_{S^3}\mathrm{CycExt}(S^4/S^1) \\
& \searrow \quad S^3 \quad \swarrow &
\end{array}
$$



Theorem (Roig-Saralegi 00) :

rationally, a direct summand of $\Omega^\infty_{S^3}\Sigma^\infty_{S^3}(S^4/S^1)$ is twisted connective K-theory $\mathrm{ku}/_{B^2\mathbb{Z}}$



and now there is a lift:
the unified cocyle of all the type IIA D-branes

dgc-algebra for $B^3\mathbb{Z} \simeq_{\mathbb{Q}} S^3$: $dH_3 = 0$
dg-module for $\mathrm{ku}/B^2\mathbb{Z}$: $dF_{2p+2} = H_3 \wedge F_{2p}$ $\quad p \in \mathbb{N}$

$$\mathbb{T}^{9,1|\mathbf{16}+\overline{\mathbf{16}}} \xrightarrow{\mu^{\mathrm{IIA}}_{F1/Dp}} \mathrm{ku}/\!/_{B^2\mathbb{Z}}$$

Conclusion:

Double dimensional reduction
of unified M-brane cocycle
via cyclification
is unified IIA-brane cocycle

$$\mathbb{T}^{9,1|\mathbf{16}+\overline{\mathbf{16}}} \xrightarrow{\mu^{\mathrm{IIA}}_{F1/Dp}} \mathrm{ku}/_{B^2\mathbb{Z}} \to \mathrm{KU}/_{B^2\mathbb{Z}}$$

Conclusion:

Double dimensional reduction
of unified M-brane cocycle
via cyclification
is unified IIA-brane cocycle

$$
\begin{array}{ccc}
\mathbb{T}^{9,1|\mathbf{16}+\overline{\mathbf{16}}} & \xrightarrow{\;(\mu^{\mathrm{IIA}}_{F1/Dp})\;} & \mathrm{KU}/\!_{B^2\mathbb{Z}} \\
\| & & \\
\mathrm{Ext}_{\mathrm{IIA}}(\mathbb{T}^{8,1|\mathbf{16}+\overline{\mathbf{16}}}) & &
\end{array}
$$

we repeat the process:

and consider the double dimensional reduction of the IIA-cocycle

to 9d super-spacetime $\mathbb{T}^{8,1|\mathbf{16}+\mathbf{16}}$

$$\begin{array}{ccc} \mathrm{Cyc}\big(\mathbb{T}^{9,1|\mathbf{16}+\overline{\mathbf{16}}}\big) & \xrightarrow{\quad \mathrm{Cyc}(\mu^{\mathrm{IIA}}_{F1/Dp}) \quad} & \mathrm{Cyc}\big(\mathrm{KU}/_{B^2\mathbb{Z}}\big) \\ \Vert & & \\ \mathrm{Cyc}\mathrm{Ext}_{\mathrm{IIA}}\big(\mathbb{T}^{8,1|\mathbf{16}+\overline{\mathbf{16}}}\big) & & \end{array}$$

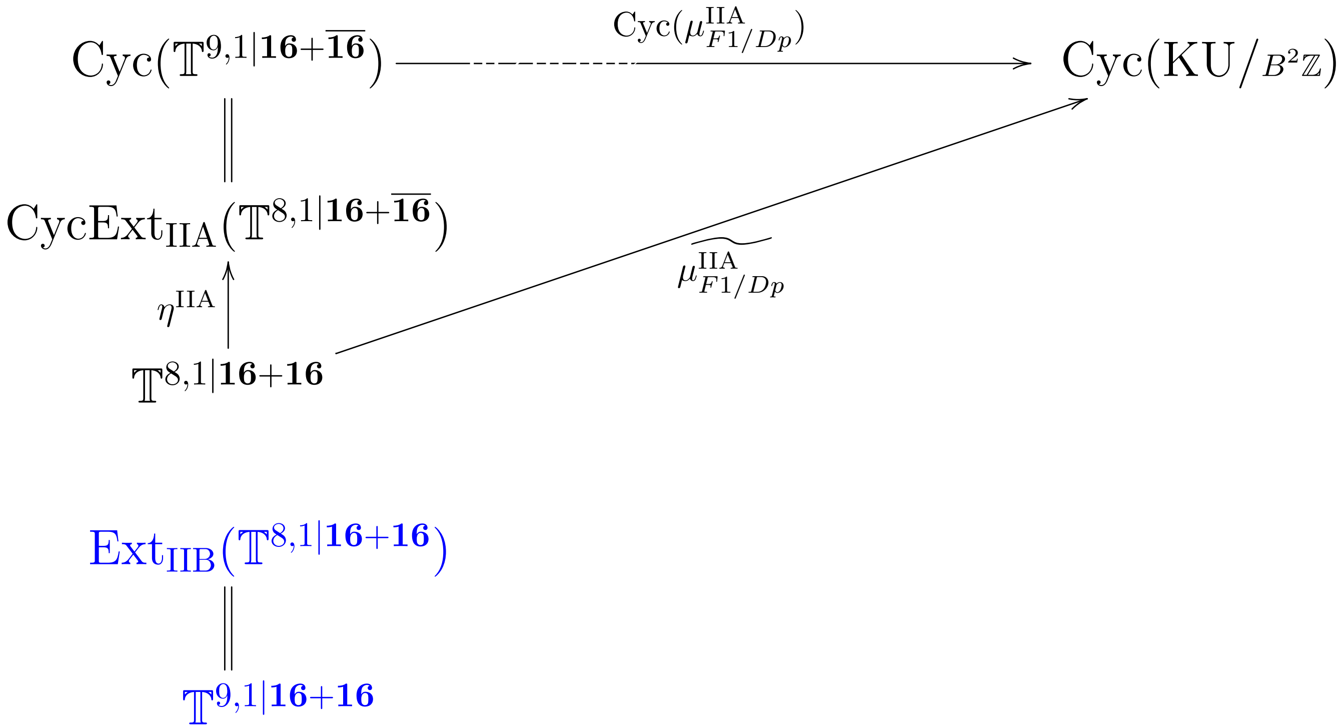
hence apply cyclification

$$
\begin{array}{ccc}
\mathrm{Cyc}\big(\mathbb{T}^{9,1|\mathbf{16}+\overline{\mathbf{16}}}\big) & \xrightarrow{\;\mathrm{Cyc}(\mu^{\mathrm{IIA}}_{F1/Dp})\;} & \mathrm{Cyc}\big(\mathrm{KU}/\!_{B^2\mathbb{Z}}\big) \\
\Big\| & & \\
\mathrm{CycExt}_{\mathrm{IIA}}\big(\mathbb{T}^{8,1|\mathbf{16}+\overline{\mathbf{16}}}\big) & & \\
\Big\uparrow {\scriptstyle \eta^{\mathrm{IIA}}} & & \\
\mathbb{T}^{8,1|\mathbf{16}+\mathbf{16}} & &
\end{array}
$$

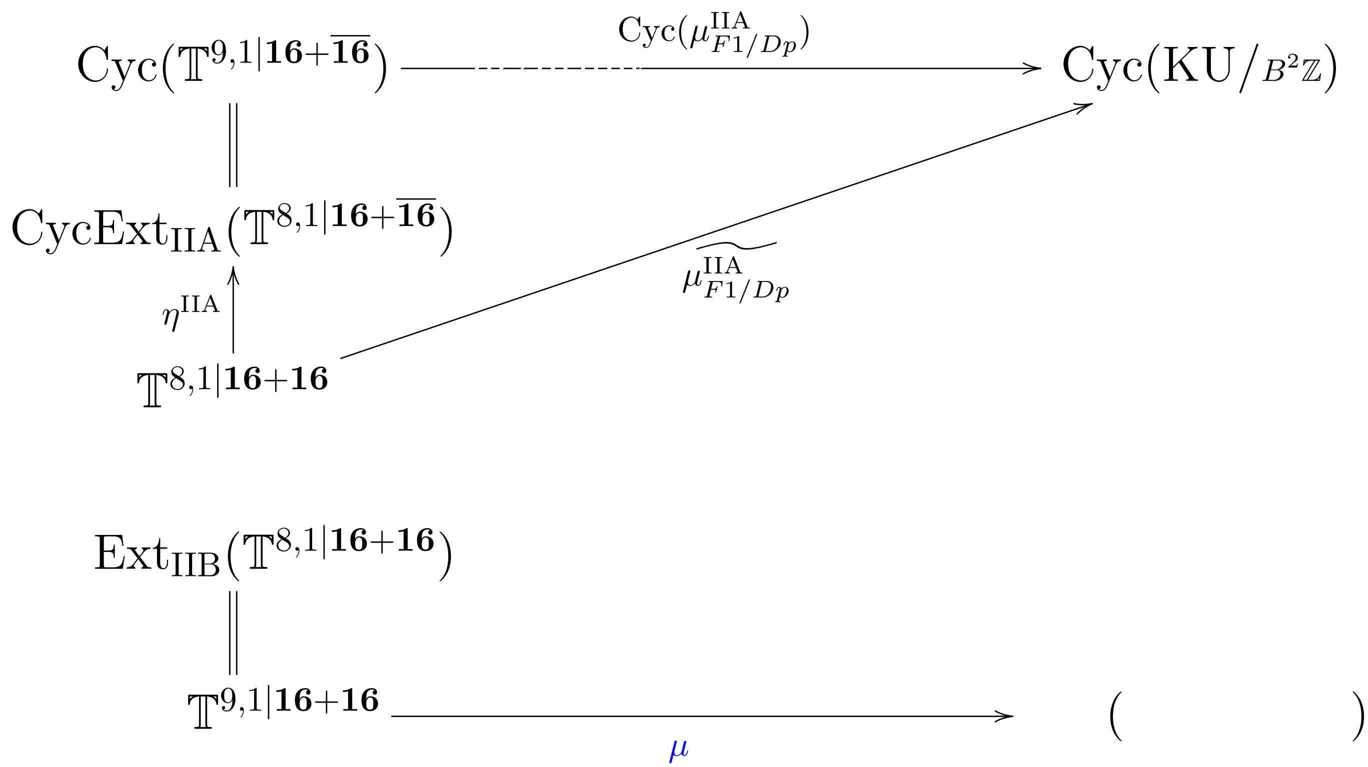
and compose
with the adjunction unit

$$
\begin{array}{ccc}
\mathrm{Cyc}\big(\mathbb{T}^{9,1\vert \mathbf{16}+\overline{\mathbf{16}}}\big) & \xrightarrow{\;\mathrm{Cyc}(\mu^{\mathrm{IIA}}_{F1/Dp})\;} & \mathrm{Cyc}\big(\mathrm{KU}/_{B^2\mathbb{Z}}\big) \\
\Vert & & \\
\mathrm{CycExt}_{\mathrm{IIA}}\big(\mathbb{T}^{8,1\vert \mathbf{16}+\overline{\mathbf{16}}}\big) & \nearrow_{\widetilde{\mu^{\mathrm{IIA}}_{F1/Dp}}} & \\
\uparrow{\scriptstyle \eta^{\mathrm{IIA}}} & & \\
\mathbb{T}^{8,1\vert \mathbf{16}+\mathbf{16}} & &
\end{array}
$$

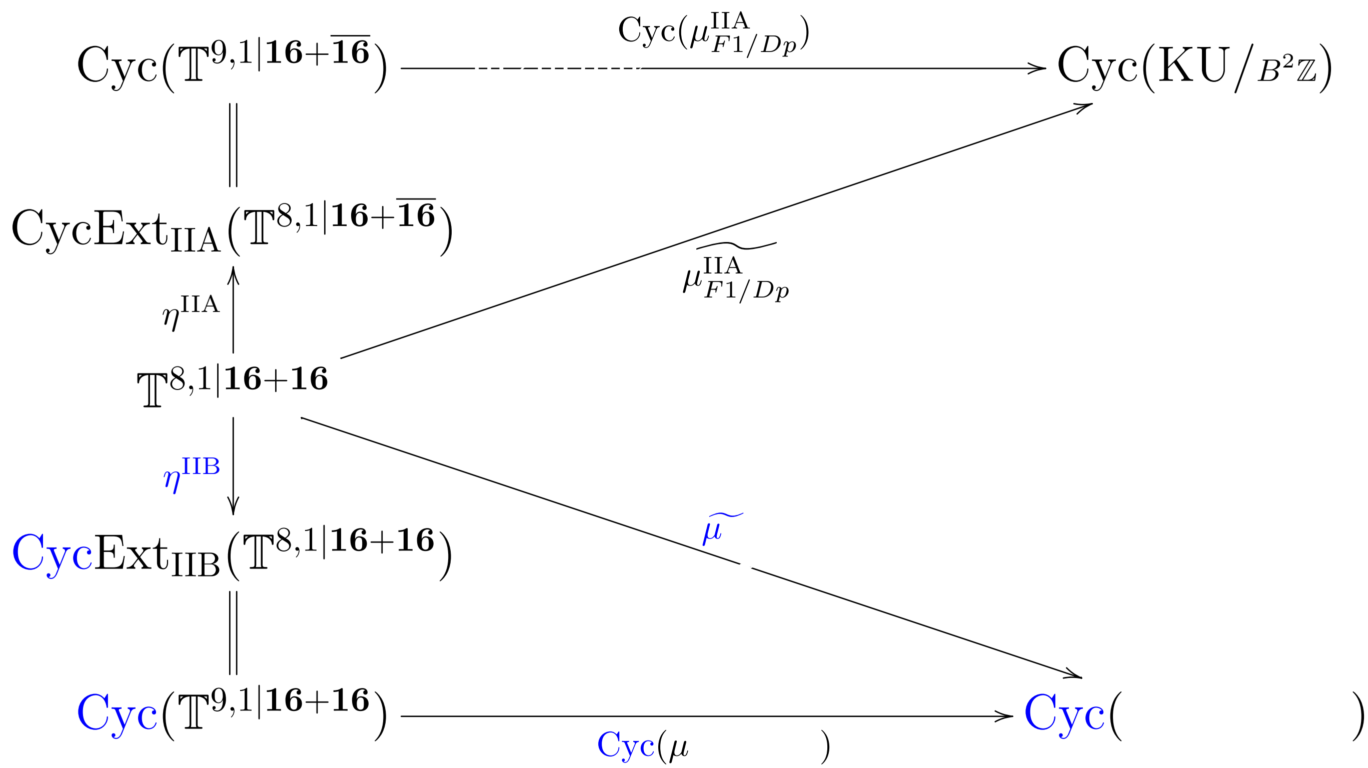
to obtain
the double dimensional reduction



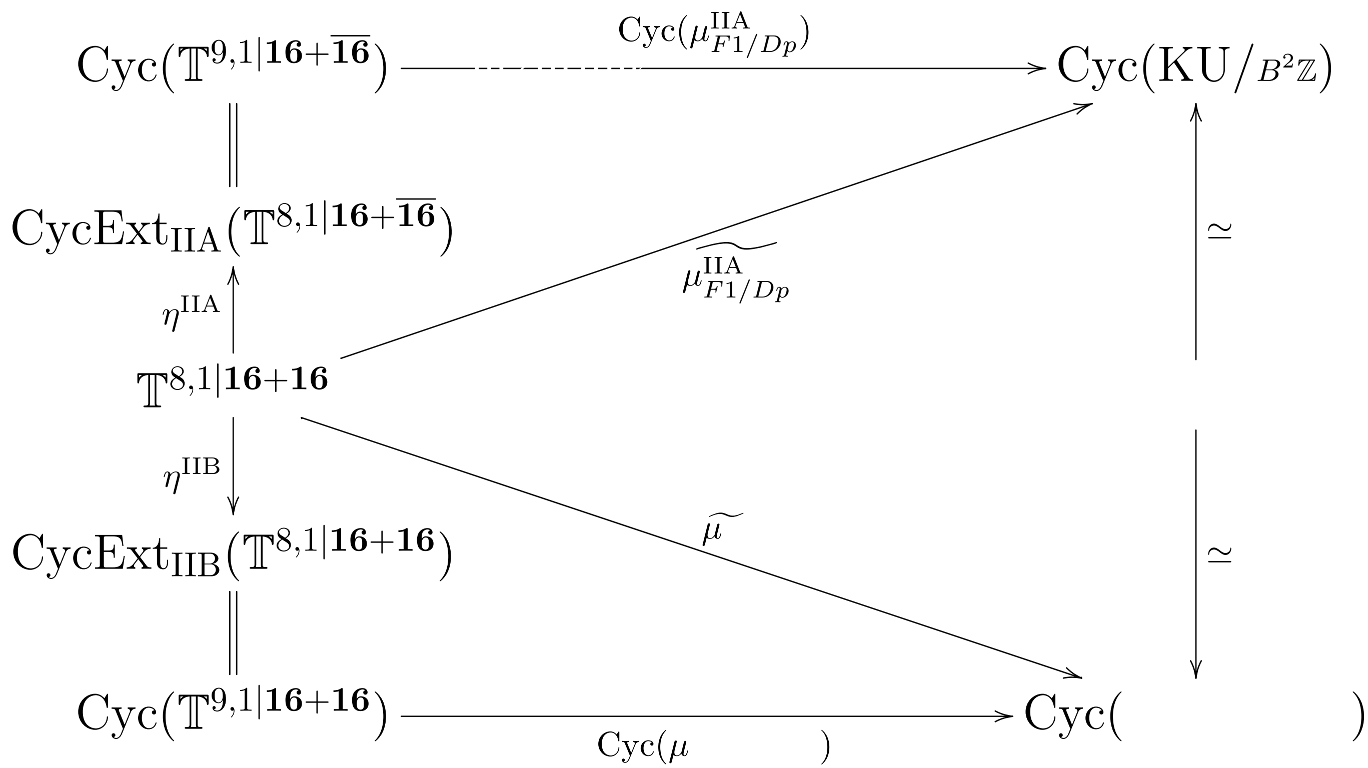
but there was also
the type IIB extension



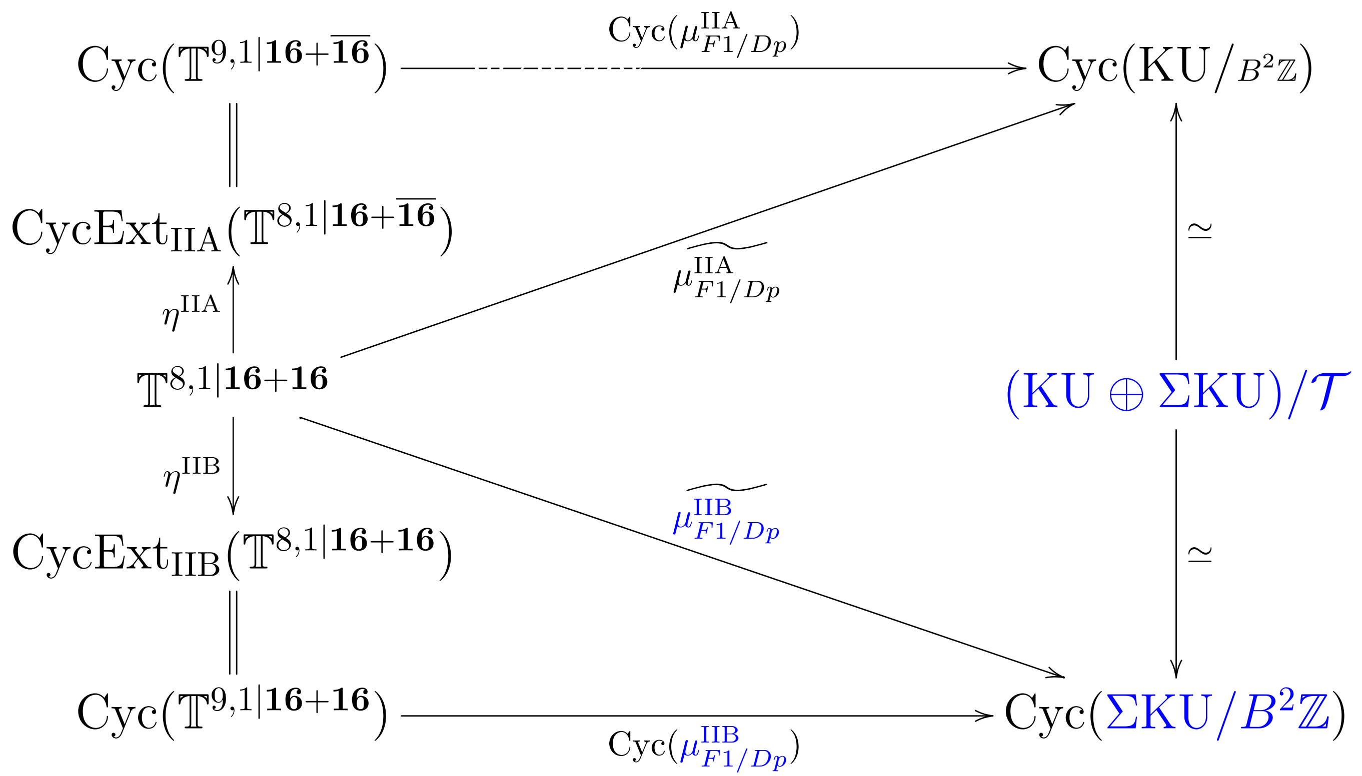
whatever cocycle it carries



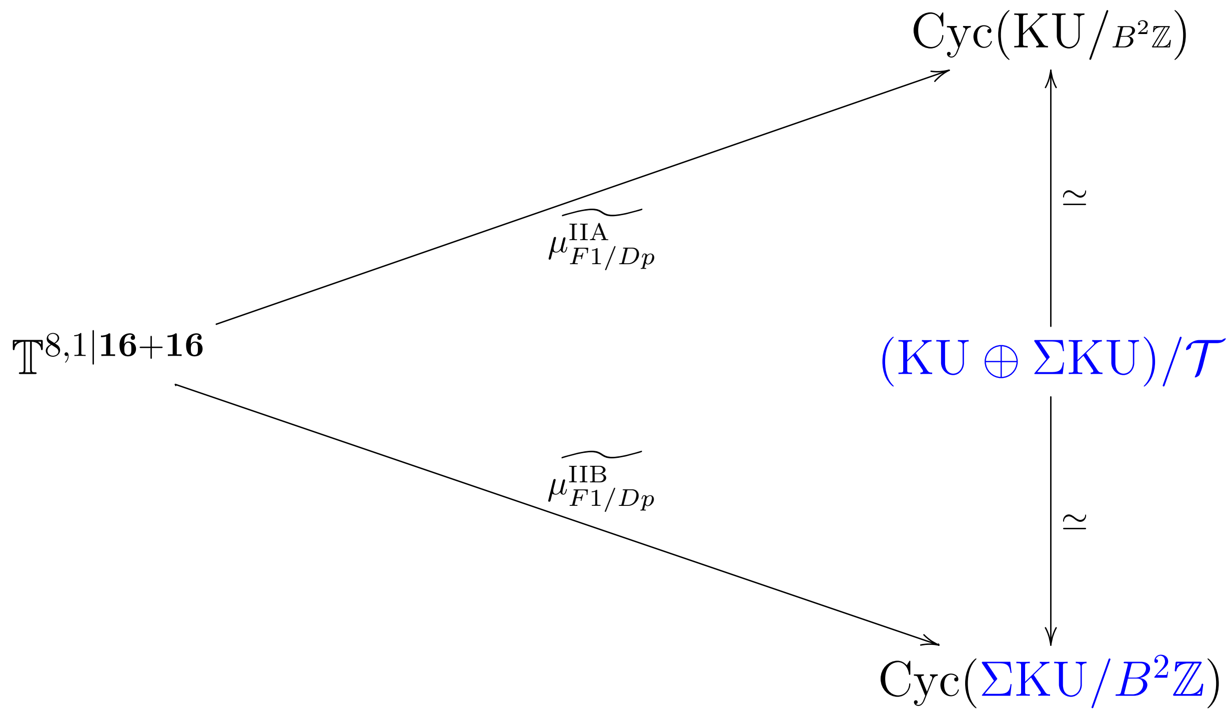
has itself a double dimensional reduction



by adjunction
this defines μ
in terms of μ^{IIA}

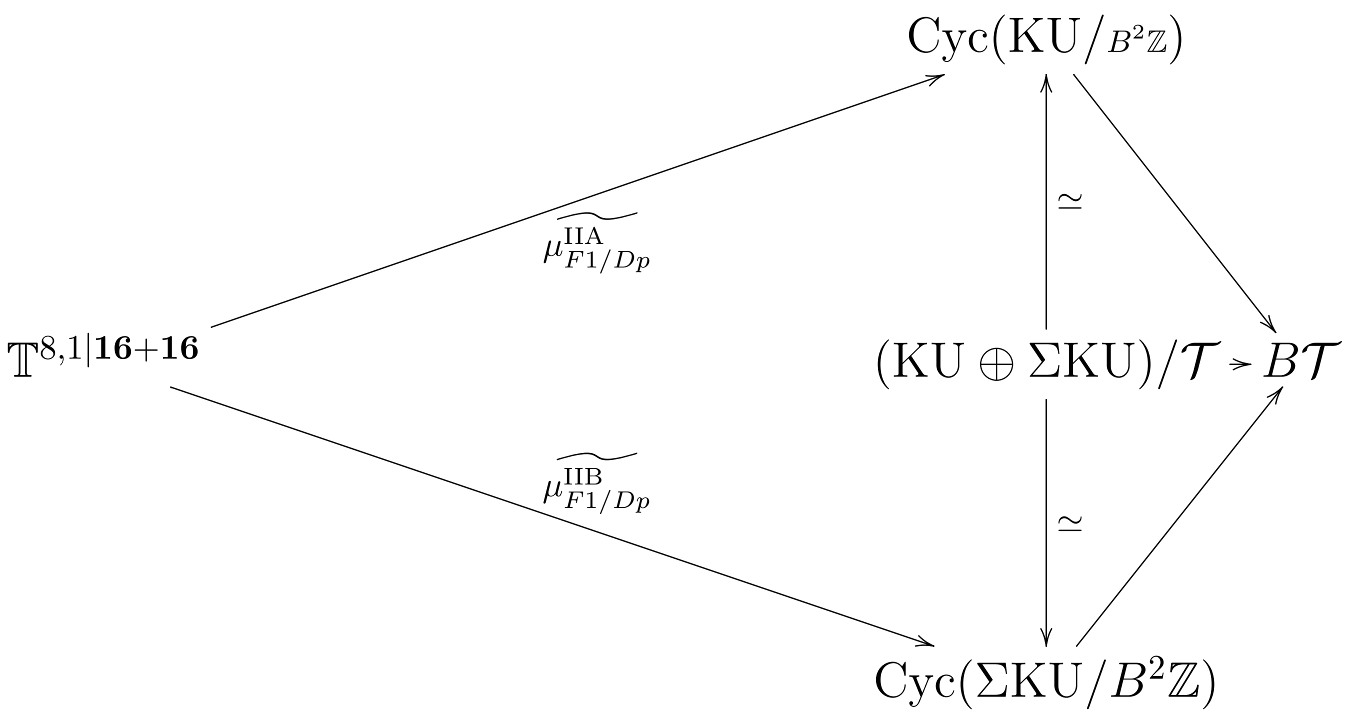


Theorem A: (Fiorenza-Sati-Schreiber 17):
This is the cocycle in twisted K^1
for the F1/Dp-branes in type IIB

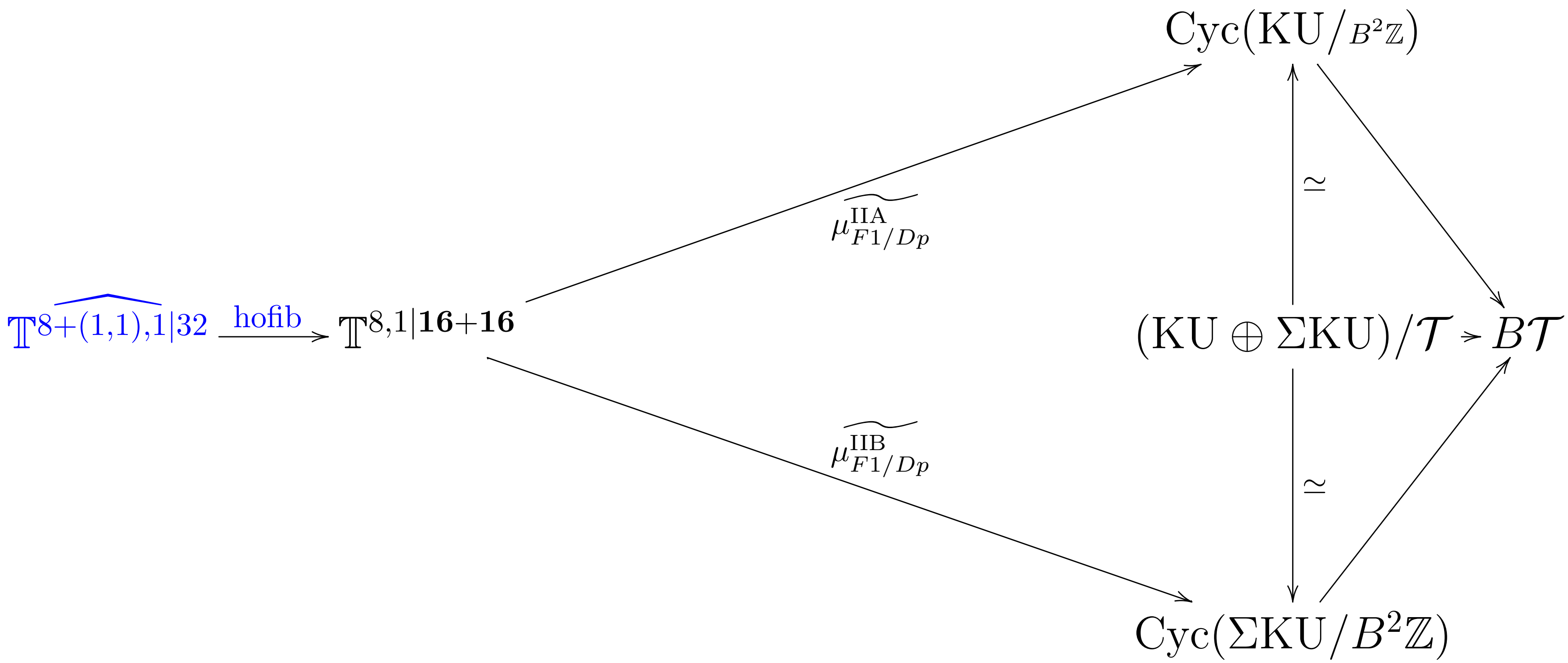


Theorem B: (Fiorenza-Sati-Schreiber 17):
The commutativity of this diagram is equivalently the Buscher rules for the RR-fields

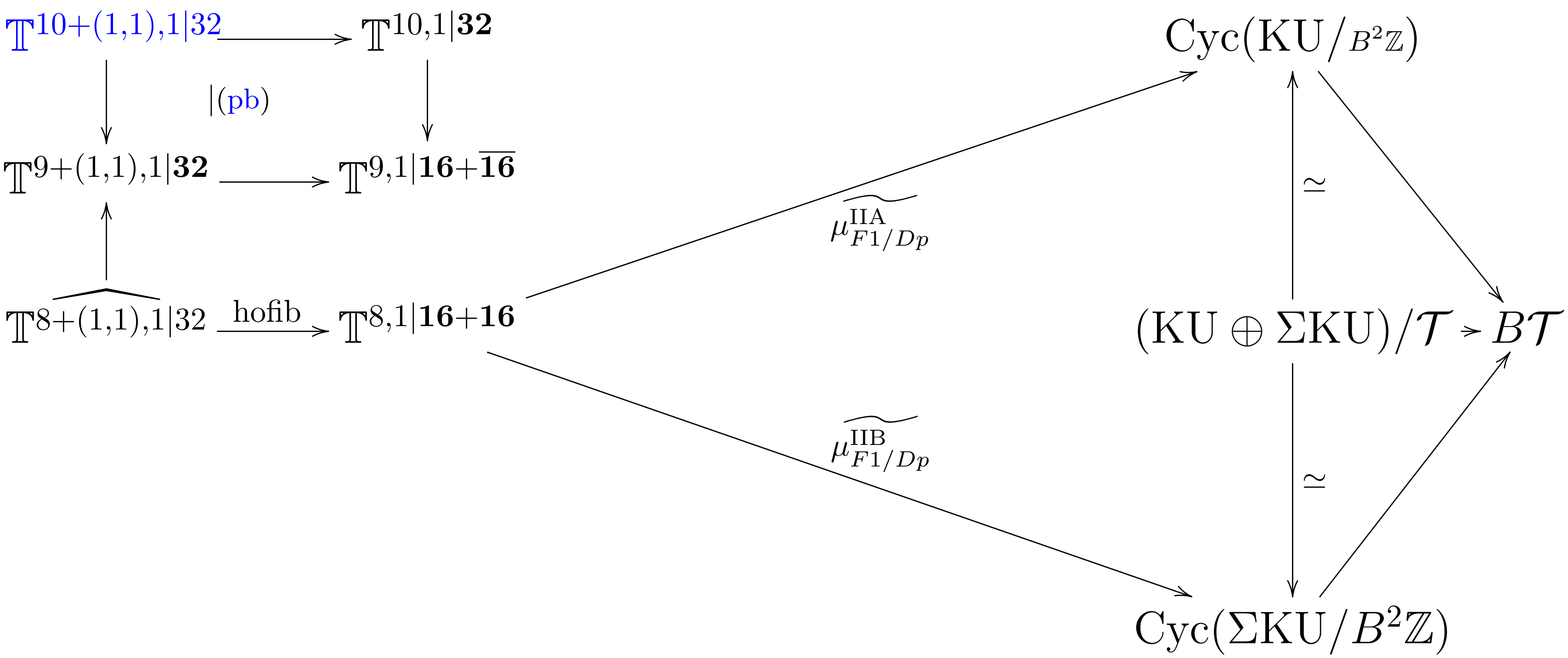
(Hori 99)



Theorem C: (Fiorenza-Sati-Schreiber 17):
The commutativity of this diagram is equivalently
the rules of "topological T-duality"
(Bouwknegt-Evslin-Mathai 04, Bunke-Rumpf-Schick 08)
rationally



Theorem D: (Fiorenza-Sati-Schreiber 17):
The homotopy fiber
is the doubled
generalized geometry
10d super-spacetime



Theorem E: (Fiorenza-Sati-Schreiber 17):
The homotopy pullback
of type II doubled super-spacetime
back to 11d super-spacetime
is the local model for an F-theory fibration

Conclusion:

A fair bit of
the expected structure of M-theory
emerges out of the superpoint
in rational super-homotopy theory.

Evident Conjecture:

The full theory emerges
once passing beyond the rational approximation
in full super-geometric homotopy theory.
(arXiv:1310.7930).

Epilogue

In full super-geometric homotopy theory
the superpoint $\mathbb{R}^{0|1}$ itself
emerges from $\emptyset$

$$
\begin{array}{ccccccccc}
\mathrm{id} & \dashv & \mathrm{id} & & & & & & \\
\vee & & \vee & & & & & & \\
\rightrightarrows & \dashv & \rightsquigarrow & \dashv & \boxed{\mathbb{R}^{0|1}} & & & & \\
 & & \vee & & \vee & & & & \\
 & & \Re & \dashv & \boxed{\mathbb{D}} & \dashv & \text{Et} & & \\
 & & & & \vee & & \vee & & \\
 & & & & \boxed{\mathbb{R}} & \dashv & \flat & \dashv & \sharp \\
 & & & & & & \vee & & \vee \\
 & & & & & & \emptyset & \dashv & *
\end{array}
$$

(Schreiber 16, FOMUS proceedings)