4d $N = 4$ SYM with Varying Coupling

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4d $N = 4$ SYM

4d $N = 4$ SYM with gauge group G is conjectured to have an $SL_2\mathbb{Z}$ Montonen-Olive duality:

$$\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2} = \tau_1 + i\tau_2\,, \qquad \tau \to \frac{a\tau + b}{c\tau + d}\,.$$

Under S-duality: G maps to the Langlands dual group $G^\vee$.

For $G = U(1)$:

$$S_{SYM} = \frac{1}{4\pi}\int (\tau_2 * F \wedge F - i\tau_1 F \wedge F + *d\phi \wedge d\phi) + \frac{i}{2\pi}\int \tilde{\lambda}\partial\!\!\!/\lambda$$

Symmetry group: $SO(1,3)_L \times SU(4)_R$

$$A_\mu \;:\; (\mathbf{2},\mathbf{2};\mathbf{1}) \qquad \phi_i \;:\; (\mathbf{1},\mathbf{1};\mathbf{6}) \qquad \lambda \;:\; (\mathbf{2},\mathbf{1};\mathbf{4}) \qquad \tilde{\lambda} \;:\; (\mathbf{1},\mathbf{2};\bar{\mathbf{4}})$$

Supersymmetries:

$$Q : \quad (\mathbf{2},\mathbf{1};\bar{\mathbf{4}})$$

$$\widetilde{Q} : \quad (\mathbf{1},\mathbf{2};\mathbf{4})$$

Standard setup: τ constant along the 4d spacetime.

What happens when we allow τ to vary in spacetime?

Variation of τ has to be consistent with the duality

$\tau \to -1/\tau$ type monodromies in spacetime along loci where τ is singular

Geometrize: τ identified with complex structure of an elliptic curve $\Rightarrow$ F-theory

String Theory Embedding

4d $N = 4$ SYM is the theory on D3-branes in IIB 10d string theory.

IIB strings also have a self-duality $SL_2\mathbb{Z}$, acting on the complexified string coupling ”axio-dilaton”

$$\tau_{IIB} = C_0 + ie^{-\phi}$$

This duality descends on D3-branes to Montonen-Olive duality

In IIB, the ”varying τ_{IIB}” version is F-theory. [Vafa][Morrison, Vafa]

$\Rightarrow$ consider D3-branes in F-theory with varying τ.

Two setups:

- D3-instantons: τ varies over full 4d spacetime M_4 [Martucci][Assel, SSN]
- ”Strings”: $M_4 = C \times \mathbb{R}^{1,1}$, τ varies over C [Lawrie, SSN, Weigand]

More Motivations: Wrapped D3-branes in F-theory

F-theory is IIB with varying $\tau = C_0 + ie^{-\phi}$.
On elliptically fibered Calabi-Yau Y_n: minimal susy

$$\begin{array}{ccl} \mathbb{E}_\tau & \hookrightarrow Y_n & \\ & \downarrow & \\ & B_{n-1} \supset C & \leftarrow \ \tau \text{ varies over } C\,. \end{array}$$

D3-branes in 6d (CY3) and 2d (CY5) F-theory compactifications:

6d: Classification of 6d (1,0) SCFTs from F-theory on CY three-folds

[Heckman, Morrison, Vafa]

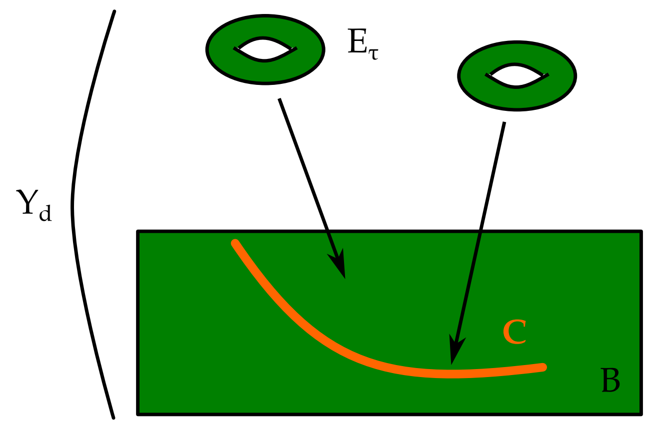
→ tensionless strings are diagnostic for superconformal invariance

[Haghighat, Klemm, Vafa, del Zotto, Lockhart, ...]

2d: (0,2) F-theory vacua [SSN, Weigand][Apruzzi, Heckman, Hassler, Melnikov]

→ D3s for tadpole cancellation $[C_{D3}] = \frac{1}{24} c_4(CY_5) - \frac{1}{2} G_4 \wedge G_4$

Cartoon of Setup:



Y_d is an elliptically fibered CY d-fold, with section

$$y^2 = x^3 + fx + g\,,$$

f, g sections of K_B^{-4} and K_B^{-6}, respectively.
D3-branes/$N = 4$ SYM on $C \times \mathbb{R}^{1,1}$.

Questions:

- # How to characterize wrapped D3-branes in F-theory on $C \times \mathbb{R}^{1,1}$?
- # What is the 2d SCFTs on $\mathbb{R}^{1,1}$?
- # Compute central charges, elliptic genus, etc for these 2d SCFTs – comparison with AdS dual.

Plan

I. 4d $N = 4$ SYM and Duality Twist

II. 6d point of view and Duality Defects

III. New chiral 2d $(0, 2)$ SCFTs and AdS duals

I. 4d $N=4$ SYM and Topological Duality Twist

D3-branes in IIB vs. F-theory

D3-brane effective theory: N=4 SYM with $\tau = \frac{\theta}{2\pi} + i\frac{4\pi}{g^2} = \tau_1 + i\tau_2$

Symmetry group of N=4 SYM:

$$SO(1,3)_L \times SU(4)_R$$

Supercharges

$$Q: \quad (\mathbf{2}, \mathbf{1}; \bar{\mathbf{4}})$$

$$\widetilde{Q}: \quad (\mathbf{1}, \mathbf{2}; \mathbf{4})$$

And field content:

$$A_\mu \ : (\mathbf{2}, \mathbf{2}; \mathbf{1}) \qquad \phi_i \ : (\mathbf{1}, \mathbf{1}; \mathbf{6}) \qquad \lambda \ : (\mathbf{2}, \mathbf{1}; \mathbf{4}) \qquad \tilde{\lambda} \ : (\mathbf{1}, \mathbf{2}; \bar{\mathbf{4}})$$

Consider theory on curved manifold, e.g. a curve C. To preserve susy: require topological twist.

Type IIB, const τ: D3s on $\mathbb{R}^{1,1} \times C \subset$CY [Bershadsky, Johanson, Sadov, Vafa]

Decomposition of supercharges:

$$SO(1,3)_L \times SU(4)_R \rightarrow SO(1,1)_L \times U(1)_C \times SO(4)_T \times U(1)_R$$
$$Q: \quad (\mathbf{2},\mathbf{1};\bar{\mathbf{4}}) \rightarrow (\mathbf{1}_{++} \oplus \mathbf{1}_{--}) \otimes ((\mathbf{2},\mathbf{1})_{-1} \oplus (\mathbf{1},\mathbf{2})_1)$$

Topological twist: Redefine $U(1)_C$ with $U(1)_R$ to get scalar supercharges:

$$T_{\text{twist}} = \frac{1}{2}(T_C + T_R)$$

$$Q \supset (\mathbf{2},\mathbf{1})_{-1,0} \oplus (\mathbf{1},\mathbf{2})_{+1,0}$$
$$\tilde{Q} \supset (\mathbf{2},\mathbf{1})_{+1,0} \oplus (\mathbf{1},\mathbf{2})_{-1,0}$$
$\Rightarrow$ 2d $(4,4)$ supersymmetry

Scalars become sections of K_C.

BPS equations:

$$F_{z\bar{z}} - i[\phi_{\bar{z}}, \bar{\phi}_z] = 0\,, \qquad D_z\phi = D_{\bar{z}}\bar{\phi} = 0$$

$\Rightarrow$ 2d SCFT: Sigma-model into Hitchin moduli space $\mathcal{M}_H$

F-theory (varying τ): E.g. elliptic CY3: $\mathbb{E}_\tau \hookrightarrow Y_3 \to B \supset C$

$N=4$ SYM with varying coupling τ and $\tau \to \gamma\tau = \frac{a\tau+b}{c\tau+d}$ monodromy

Under $SL_2\mathbb{Z}$, Q and $\tilde{Q}$ carry a $U(1)_D$ charge [Intriligator][Kapustin, Witten]

$$\begin{aligned} Q &\to e^{-\frac{i}{2}\alpha(\gamma)} Q \\ \widetilde{Q} &\to e^{+\frac{i}{2}\alpha(\gamma)} \widetilde{Q} \end{aligned} \qquad \text{where} \quad e^{i\alpha(\gamma)} = \frac{c\tau+d}{|c\tau+d|}\,, \quad \gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2\mathbb{Z}$$

Fields of abelian $N=4$ SYM transform as: ϕ^i invariant and

$$\begin{aligned} \lambda &\to e^{-\frac{i}{2}\alpha(\gamma)} \lambda \\ \tilde{\lambda} &\to e^{+\frac{i}{2}\alpha(\gamma)} \tilde{\lambda} \end{aligned}$$

$$F^{(\pm)}_{\mu\nu} = \frac{\sqrt{\tau_2}}{2}(F \pm \star F) \to e^{\mp i\alpha(\gamma)} F^{(\pm)}_{\mu\nu}$$

Topologial duality twist:

$U(1)_D$ and $U(1)_C$ twisted with $U(1)_R \subset SU(4)_R$ [Martucci][Assel, SSN]

Fields transform as sections of $\mathcal{L}_D = K_B^{-1}|_C$, with connection

$$\mathcal{A}_D = \frac{d\tau_1}{2\tau_2}$$

Duality Twist on $\mathbb{R}^{1,1} \times C$

Under $G_{\text{total}} = SO(1,3)_L \times SU(4)_R \times U(1)_D$:
$Q : (\mathbf{2},\mathbf{1},\bar{\mathbf{4}})_{+1}$ and $\tilde{Q} : (\mathbf{1},\mathbf{2},\mathbf{4})_{-1}$
Twist on $C \subset B \subset CY_3$: as before

$$SO(1,3)_L \to SO(1,1)_L \times U(1)_C \qquad SU(4)_R \to SO(4)_T \times U(1)_R$$

Duality Twist:

$$T_C^{\text{twist}} = \frac{1}{2}(T_C + T_R) \qquad T_D^{\text{twist}} = \frac{1}{2}(T_D + T_R)\,.$$

$$\begin{array}{rcl} G_{\text{total}} & \to & SO(4)_T \times SO(1,1)_L \times U(1)_C^{\text{twist}} \times U(1)_D^{\text{twist}} \\ Q = (\mathbf{2},\mathbf{1},\bar{\mathbf{4}})_{+1} & \to & \underline{(\mathbf{2},\mathbf{1})_{1;0,0}} \oplus (\mathbf{2},\mathbf{1})_{-1;-1,0} \oplus (\mathbf{1},\mathbf{2})_{1;1,1} \oplus (\mathbf{1},\mathbf{2})_{-1;0,1} \\ \tilde{Q} = (\mathbf{1},\mathbf{2},\mathbf{4})_{-1} & \to & \underline{(\mathbf{2},\mathbf{1})_{1;0,0}} \oplus (\mathbf{2},\mathbf{1})_{-1;1,0} \oplus (\mathbf{1},\mathbf{2})_{1;-1,-1} \oplus (\mathbf{1},\mathbf{2})_{-1;0,-1} \end{array}$$

$\Rightarrow$ $(0,4)$ SUSY in 2d $\mathbb{R}^{1,1}$ (remaining 4 supercharges are broken by transformations under $U(1)_D$)

Duality twists for D3-branes in CY_n

Amount of susy in 2d depends on specific duality twist, which in turn depends on the geometry:

$$SU(4)_R \quad \rightarrow \quad \begin{cases} SU(4)_R & \text{CY}_2 \text{ Duality twist: } (0,8) \\ SO(4)_T \times \underline{U(1)_R} & \text{CY}_3 \text{ Duality twist: } (0,4) \\ SU(2)_R \times SO(2)_T \times \underline{U(1)_R} & \text{CY}_4 \text{ Duality twist: } (0,2) \\ SU(3)_R \times \underline{U(1)_R} & \text{CY}_5 \text{ Duality twist: } (0,2) \end{cases}$$

NB:

All duality twisted 2d models are chiral

K3 requires only one twist $T_{\text{twist}} = \frac{1}{2}(T_C + T_D)$

Spectrum of 2d Strings

Decompose abelian 4d $N = 4$ matter with respect to the topological duality twist: and identify fields as bundle-valued forms. E.g. CY4-Duality Twist:

$$
\begin{aligned}
G_{\text{total}} \quad &\rightarrow \quad SU(2)_R \times U(1)_C^{\text{twist}} \times U(1)_D^{\text{twist}} \\
\phi^i : \ (\mathbf{1}, \mathbf{6})_0 \quad &\rightarrow \quad \mathbf{1}_{0,0} \oplus \mathbf{1}_{0,0} \oplus \mathbf{2}_{+1,+1} \oplus \mathbf{2}_{-1,-1}
\end{aligned}
$$

Identify:

$(q_C^{\text{twist}}, q_D^{\text{twist}}) = (-1, 0)$ section of K_C

$(q_C^{\text{twist}}, q_D^{\text{twist}}) = (0, -1)$ section of $\mathcal{L}_D = K_B^{-1}|_C$

$\Rightarrow \mathbf{2}_{\frac{1}{2},\frac{1}{2}}$ is a section of $N_{C/B}$ as $K_C = \mathcal{L}_D^{-1} \otimes \Lambda^2 N_{C/B}$

Spectrum of duality twisted D3 on $C \subset$ CY3

$(q_C^{\text{twist}}, q_D^{\text{twist}})$	Fermions		Bosons		$(0,4)$	Multiplicity
$(1,1)$ $(-1,-1)$	$(\mathbf{2},\mathbf{1})_1$ $(\mathbf{2},\mathbf{1})_1$	ψ_+ $\tilde{\psi}_+$	$(\mathbf{1},\mathbf{1})_0$, $(\mathbf{1},\mathbf{1})_0$ $(\mathbf{1},\mathbf{1})_0$, $(\mathbf{1},\mathbf{1})_0$	$\bar{a}, \bar{\phi}$ a, ϕ	Hyper	$h^0(C, K_C \otimes \mathcal{L}_D)$ $= g - 1 + c_1(B_2) \cdot C$
$(0,0)$	$(\mathbf{1},\mathbf{2})_1$ $(\mathbf{1},\mathbf{2})_1$	μ_+ $\tilde{\mu}_+$	$(\mathbf{2},\mathbf{2})_0$	φ	Twisted Hyper	$h^0(C) = 1$
$(1,0)$ $(-1,0)$	$(\mathbf{1},\mathbf{2})_{-1}$ $(\mathbf{1},\mathbf{2})_{-1}$	$\tilde{\rho}_-$ ρ_-			Fermi	$h^1(C) = g$
$(0,1)$ $(0,-1)$	$(\mathbf{2},\mathbf{1})_{-1}$ $(\mathbf{2},\mathbf{1})_{-1}$	λ_- $\tilde{\lambda}_-$	$(\mathbf{1},\mathbf{1})_2$ $(\mathbf{1},\mathbf{1})_{-2}$	v_+ v_-	Vector	$h^1(C, K_C \otimes \mathcal{L}_D) = 0$

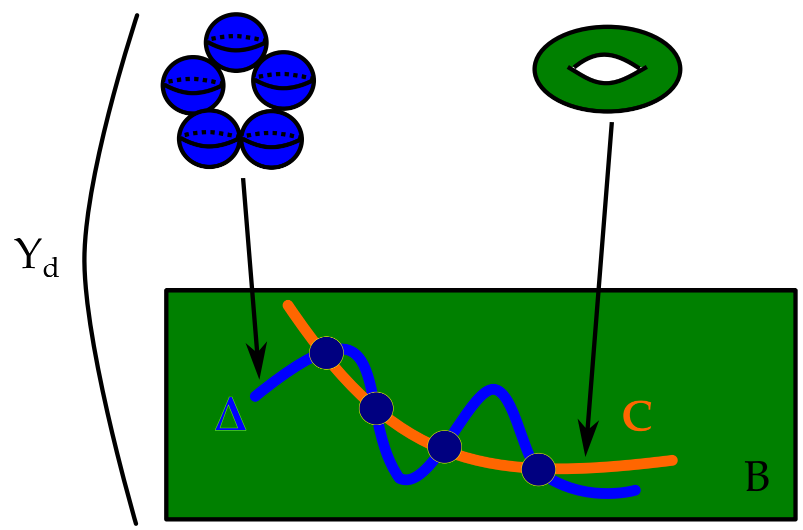
Note: this is for the abelian $N = 4$ SYM.

Key omission so far: Singular Fibers

Geometrize: τ as complex structure of elliptic fibration Y_d in F-theory

$$y^2 = x^3 + fx + g$$

Elliptic fibration can becomes singular.



Δ = $4f^3 + 27g^2$ = discriminant locus (F-theory: '7-branes')

τ undergoes $SL_2\mathbb{Z}$ monodromy around $\Delta = 0$ loci.

Duality Defects at $C \cap \Delta$ characterized by Kodaira singular fibers
[In F-theory: at intersections with 7-branes get 3-7 open strings]

BPS-equations and Hitchin moduli space

For τ constant, $N = 4$ SYM on $C \times \mathbb{R}^{1,1}$: 2d $(4,4)$ susy sigma-model into the Hitchin moduli space [Bershadsky, Johansen, Sadov, Vafa][Kapustin, Witten]

In duality-twisted theories the BPS equations along C are [Lawrie, SSN]

$$(*) \qquad \begin{aligned} \mathcal{F}_{\mathcal{A}} - i[\overline{\phi}, \phi] &= 0 \\ D_{\mathcal{A}}\overline{\phi} = \overline{D}_{\mathcal{A}}\phi &= 0 \end{aligned}$$

Duality-twisted Hitchin equations, with

$$\mathcal{F}_{\mathcal{A}} \equiv \frac{1}{2}(\overline{D}_{\mathcal{A}} a - D_{\mathcal{A}} \overline{a})\,, \qquad \mathcal{A} = \frac{d\tau_2}{2\tau_1}$$

where now

$$\phi, a \in H^0(K_C \otimes \mathcal{L}_D \otimes \mathcal{O}(\delta))$$

including $\delta = \Delta \cap C$ defects. Note: $\mathcal{L}_D = K_B^{-1}|_C$, and ϕ: $N_{C/B} \otimes \mathcal{O}(\delta)$.

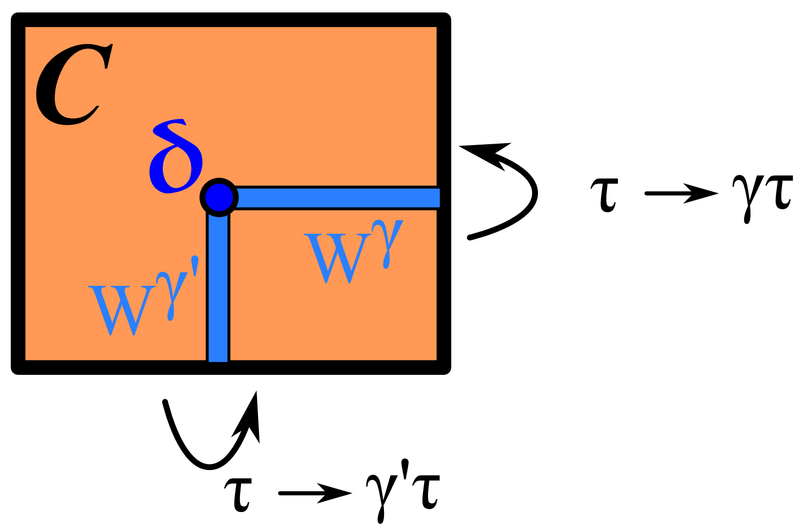
$\Rightarrow$ 4d $N = 4$ SYM on $C \times \mathbb{R}^2$ is a sigma-model into the duality twisted Hitchin moduli space for $(*)$

Alternative Description?

S-duality acts on the Hitchin moduli space as T-duality/Mirror Symmetry on the fibers of the Hitchin fibration of $\mathcal{M}_H$: setup with defects at δ

$$C \quad \to \quad \mathcal{M}_H = (T^d \to B_H)$$

where now the fiber undergoes T-duality/MS, as they encircle defects at $\delta = C \cap \Delta$: "T-fold"



$\gamma, \gamma' \in SL_2\mathbb{Z}$
W^γ are 1d walls, corresponding to the branch-cuts of τ.

Gauge theoretic description of walls and defects

Locally we can cut up $C = \cup C_i$ and W_{ij} 3d walls between these regions, where τ has a branch-cut.

Define

$$F_D = \tau_1 F + i\tau_2 \star F$$

then the action of $\gamma \in SL_2\mathbb{Z}$ monodromy on the gauge field is

$$(F_D^{(j)}, F^{(j)})\Big|_{W_{ij}} = \gamma(F_D^{(i)}, F^{(i)})\Big|_{W_{ij}}$$

This maps the gauge part $S_F = -\frac{i}{4\pi}\int_{C\times\mathbb{R}^{1,1}} F \wedge F_D$ to itself, except for an offset on the 3d wall (see also [Ganor])

$$S^{\gamma}_{W_{ij}} = -\frac{i}{4\pi}\int_{W_{ij}} \left(A^{(i)} \wedge F_D^{(i)} - A^{(j)} \wedge F_D^{(j)}\right)$$

E.g. $\gamma = T^k$ this is a level k CS term.

Theory on the Duality Defect

3-7 strings for general elliptic fibrations are poorly understood. E.g. the naive expectation that the number of defect modes is $C.[\Delta] = 12C \cdot c_1(B)$, is wrong, as not all 7-branes are in the same $SL_2\mathbb{Z}$ rep.

Other ways to access the spectrum of defects:

- # 6d point of view:
  M5-branes on elliptic surfaces $\Rightarrow$ 3-7 modes from dim redux
- # Anomalies: Strings carry chiral modes $\Rightarrow$ Anomalies
- # Holography: AdS dual to 2d SCFTs computes central charges

II. 6d Point of View and Duality Defects

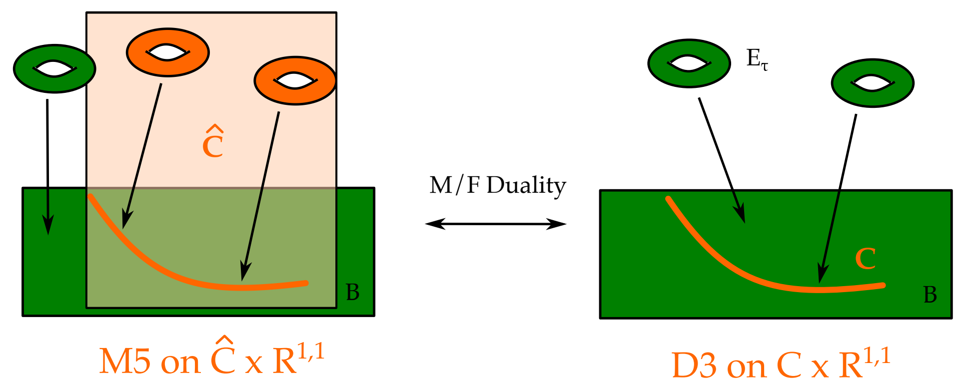
6d Point of View

Unique 6d SCFT with ADE gauge group and $(2,0)$ supersymmetry:

$\{$6d $(2,0)$ theory on $\mathbb{E}_\tau \times \mathbb{R}^{1,3}\}$ = $\{N=4$ SYM on $\mathbb{R}^{1,3}$ with coupling τ $\}$

Generalization:

$\{$6d $(2,0)$ theory on elliptic fibration$\}$ = $\{$ 4d $N=4$ SYM with varying $\tau\}$



$\widehat{C} = (\mathbb{E}_\tau \to C)$ is obtained from restricting the elliptic fibration to C.

Standard Topological Twist on Kähler manifold $\widehat{C}$ of the 6d (2,0) theory
= Topological duality twist of 4d N=4

Advantage: can be generalized to non-abelian theory, includes defect modes [Assel, SSN]

The 6d $(2,0)$ Theory

Lorentz and R-symmetry:

$$SO(6)_L \times Sp(4)_R \subset OSp(6|4)$$

Tensor multiplet:

$$\begin{aligned} \mathcal{B}_{MN}: &\qquad (\mathbf{15},\mathbf{1}) \qquad \text{with selfduality } \mathcal{H} = d\mathcal{B} = *_6\mathcal{H} \\ \Phi^{\widehat{m}\widehat{n}}: &\qquad (\mathbf{1},\mathbf{5}) \\ \rho^{\widehat{m}}: &\qquad (\bar{\mathbf{4}},\mathbf{4}) \end{aligned}$$

Abelian EOMs:

$$\mathcal{H}^- = d\mathcal{H} = 0\,, \qquad \partial^2\Phi^{\widehat{m}\widehat{n}} = 0\,, \qquad \not{\partial}\rho^{\widehat{m}} = 0\,.$$

6d $(2,0)$ Theory on Elliptic Surface $\widehat{C}$

Symmetries: $SO(1,5)_L \times Sp(4)_R \subset OSp(6|4)$

Standard topological twist:

$$SO(1,5)_L \to SO(1,1)_L \times SU(2)_\ell \times U(1)_\ell : \qquad \mathbf{4} \to \mathbf{2}_{0,1} \oplus \mathbf{1}_{1,-1} \oplus \mathbf{1}_{-1,-1}$$
$$Sp(4)_R \to SU(2)_R \times U(1)_R : \qquad \mathbf{4} \to \mathbf{2}_1 \oplus \mathbf{2}_{-1}$$

Twist on Kähler surface: $N = (0,4)$, cf. [Maldacena, Strominger, Witten]

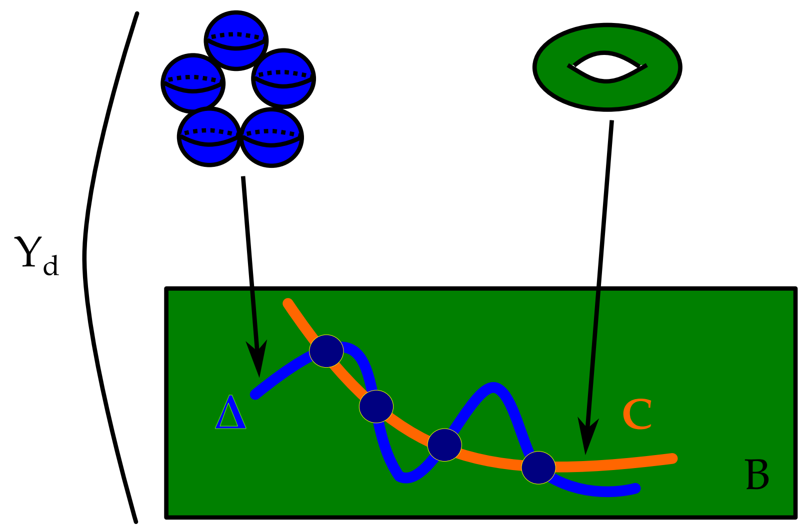
$$T_{U(1)_{\text{twist}}} = T_{U(1)_\ell} + T_{U(1)_R}$$

Specializing to an elliptic Kähler surface $\widehat{C}$, with base C. Fibration:

$$\omega^{\mathbb{E}_\tau} = \frac{d\tau_1}{2\tau_2} = \mathcal{A}_D$$

Thus: $T_{U(1)_\ell} = T_{U(1)_C} + T_{U(1)_D}$ and the top twist for the M5-brane on Kähler surface becomes topological duality twist and can be generalized to non-abelian case. [Assel, SSN]

Including Singular Fibers



Singular fibers:
additional $\omega^i_{(1,1)}$ from rational curves in Kodaira fibers

$$\mathcal{H} = d\mathcal{B} = \sum_{i=1}^{k-1} \left(\partial_z b_i dz \wedge \omega^i_{(1,1)} + \partial_{\bar{z}} b_i d\bar{z} \wedge \omega^i_{(1,1)} \right) \stackrel{!}{=} *\mathcal{H}$$

$\Rightarrow$ chiral modes b_i localized along $C \cap \Delta$ (and $\mathbb{R}^{1,1}$)
$\Rightarrow$ global (flavor) symmetry, induced by the type of codim 1 singular fibers

Spectrum of 2d $(0,4)$ from M5 on $\widehat{C} \subset CY_3$

Zero-modes counted in terms of self-intersection of C in B_2 and intersection with $c_1(B_2)$, where B_2= base of the CY$_3$ elliptic fibration

Multiplicity	$(0,4)$ Multiplet	Complex scalars	R-Weyl	L-Weyl
$h^{0,0}(\widehat{C}) = 1$	Hyper	2	2	–
$h^{0,1}(\widehat{C}) = \frac{1}{2}(C \cdot C - c_1(B_2) \cdot C)$	Fermi	–	–	2
$h^{0,2}(\widehat{C}) = \frac{1}{2}(C \cdot C + c_1(B_2) \cdot C)$	Hyper	2	2	–
$h^{1,1}(\widehat{C}) - 2h^{0,2}(\widehat{C}) - 2 = 8\, c_1(B_2) \cdot C$	half-Fermi	–	–	1

Central Charges

Direct computation from 6d $(2,0)$ on the elliptic surface $\widehat{C} = \mathbb{E}_\tau \to C$ times $\mathbb{R}^{1,1}$:

$$c_R = 3C \cdot C + 3c_1(B) \cdot C + 6$$

$$c_L = 3C \cdot C + 9c_1(B) \cdot C + 6$$

From spectrum of one M5 [MSW][Vafa][Minsian,Moore][Lawrie, SSN, Weigand]

Matches with duality twisted $N = 4$ SYM except for extra Fermi multiplets, which precisely account for the defect modes

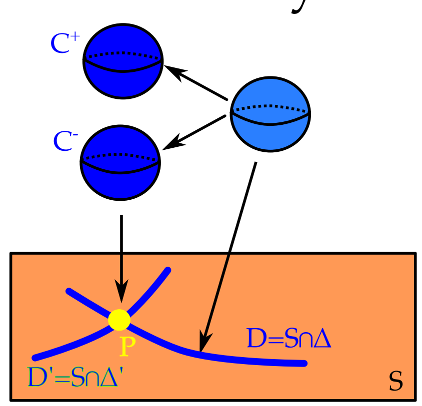
$$\delta c_L^{\text{defect}} = 8c_1(B) \cdot C$$

Generalization: intersecting defects

In models where τ varies over the full 4d spacetime S, the duality defects are curves, these chiral supersymmetric defects $\mathcal{D}$ intersect at points

$$P_{\alpha\beta} = \mathcal{D}_\alpha \cap \mathcal{D}_\beta = S \cap \Delta_\alpha \cap \Delta_\beta$$

Geometrically: Kodaira fiber $\mathbb{P}^1$s become further reducible $\mathbb{P}^1_i \to C_+ + C_-$



Duality defects form network and at intersections:

$$\left(\int_{C^+} + \int_{C^-}\right)\mathcal{B} = \int_{\mathbb{P}^1_i} \mathcal{B} \quad \to \quad b_+ + b_- = b_i$$

⇒ correspond to flavor symmetry enhancement

⇒ Description: resolved Tate model for elliptic fibration incl codim 2 fibers [Katz, Morrison, SSN, Sully][Hayashi, Lawrie, Morrison, SSN]

Strings in 4d and 2d F-theory Compactifications

CY_4 Duality twist $N = (0,2)$:

$$
\begin{aligned}
c_R =& 3(g + c_1(B_3) \cdot C + h^0(C, N_{C/B_3})) \\
c_L =& 3(g + h^0(C, N_{C/B_3})) + c_1(B_3) \cdot C + 8c_1(B) \cdot C
\end{aligned}
$$

CY_5 Duality twist $N = (0,2)$: No M5 picture, but M2

$$
\begin{aligned}
c_L &= 3(g + h^0(C, N_{C/B_4}) - 1) + 9c_1(B_4) \cdot C \\
c_R &= 3(g + c_1(B_4) \cdot C + h^0(C, N_{C/B_4}) - 1)
\end{aligned}
$$

Application to 2d $(0,2)$ vacua from CY_5 compactifications of F-theory [SSN, Weigand], [Apruzzi, Hassler, Heckman, Melnikov]. Tadpole cancellation requires D3-branes wrapped on curves in the class (for $G_4 = 0$)

$$
C = \frac{1}{24} c_4(Y_5)|_{B_4}
$$

Non-Abelian Generalization?

Lets take stock:

- For N=4 SYM with $G = U(1)$ we have a complete description of the spectrum, and central charges for duality twisted compactifications to 2d $\Rightarrow$ new chiral 2d SCFTs
- BPS equations are duality twisted Hitchin equations
- 6d point of view: useful to get defect modes (from the geometry of the singular fibers).

However: spectrum computations restricted to $G = U(1)$

Physicists:
SCFTs with $U(N)$ gauge group, large N: description in terms of "holographically dual" gravity solution.

III. AdS/CFT and Duality Twisted N=4 SYM

[Couzens, Lawrie, Martelli, SSN, Wong]

AdS/CFT in a Nutshell

A superconformal field theory in d dimensions is ″dual″ to a gravity or string theory in $d+1$ dimensional AdS_{d+1} space, where the spacetime of the SCFT lives at the boundary of AdS:

$$N = 4 \text{ SYM } G = U(N) \text{ 'dual' to IIB strings in } AdS_5 \times S^5.$$

What's the point?

The gravity/string dual describes the SCFT in a particular limit, namely at strong coupling:
$\lambda = g_{YM} N^2$ is large in the gravity, small in the gauge theory descriptions.

For certain supersymmetrically protected (BPS) quantities, the two sides can be computed in either description, and compared.

AdS_3 in F-theory

Our setup:
2d SCFTs, which should have AdS_3 duals in Type IIB supergravity.

Can we construct the dual AdS_3 solution, and compute e.g. central charges holographically?

$\Rightarrow$ Yes, but we need to extend the standard framework of AdS/CFT to include varying τ (i.e. F-theory rather than IIB AdS solutions)

AdS_3 dual to $(0,4)$ in F-theory

In summary: the most general F-theory solution dual to $(0,4)$ SCFTs in 2d is

$$\mathrm{AdS}_3 \times S^3/\Gamma \times (\mathbb{E}_\tau \hookrightarrow Y_3 \rightarrow B_2)\,, \qquad F^{(2)} = J_B$$

- # τ = complex structure of $\mathbb{E}_\tau$
  J_B = Kähler form on B_2, discrete $\Gamma \subset SU(2)$
- # Physical type IIB compactification space is $\mathrm{AdS}_3 \times S^3/\Gamma \times B_2$
  B_2= Kähler surface
- # B_2 constrained by the existence of an elliptic fibration with Weierstrass model, dP_n, F_n, blowups thereof or Enriques [Grassi][Gross]
- # For τ constant: reduces to well-known $AdS_3 \times S^3 \times CY_2$ solution

Properties of the Solution

$$\mathrm{AdS}_3 \times S^3 \times (\mathbb{E}_\tau \hookrightarrow Y_3 \to B_2)$$

- Supersymmetry: Killing spinors transform as **2** of $SU(2)_r \subset SO(4)_T$ acting on S^3
  $\Rightarrow$ R-symmetry is $SU(2)_r$ of the (0,4) small SCA
- Can allow also for S^3/Γ retaining $(0,4)$ supersymmetry.
- $\Gamma = \mathbb{Z}_M$: additional M KK-monopoles
  $\Rightarrow$ F-theory brane-setup in: $Y_3 \times \mathrm{TN}_M \times \mathbb{R}^{1,1}$.
  $\Rightarrow$ Special case of F-theory on CY 5-folds

Holographic Central Charges in IIB/F

- Leading order in N: by classical gravity results of Brown-Henneaux

$$c_L^{(2)} = c_R^{(2)} = 3\frac{R_{\mathrm{AdS}_3}}{2G_N^{(3)}}$$
$$= 3N^2 \frac{\mathrm{vol}(S^3)\mathrm{vol}(B_2) 32\pi^2}{\mathrm{vol}(S^3)^2} = 6N^2 \mathrm{vol}(B_2)$$

- N = 5-form flux quantum through $S^3 \times C$, $C \subset B_2$
- Computation of volume of B_2:
  # Fact: The metric on B_2 is singular (cf. Stringy Cosmic Strings [Greene, Shapere, Vafa, Yau]), as τ of the elliptic fibration can become singular. The metric on Y_3 is smooth.
  # Compute in Y_3: B_2 is a divisor (section of the fibration) and its volume is

$$\mathrm{vol}(B_2) = \frac{1}{2} \int_{Y_3} \omega_0 \wedge \pi^* J_B \wedge \pi^* J_B$$

  where $\omega_0 = (1,1)$ form dual to B_2 and $\pi : Y_3 \to B_2$.

- Algebro-geometrically: $\mathrm{vol}(B_2) = \frac{1}{2}\int_B J_B \wedge J_B = \frac{1}{2} C \cdot_B C$, where C = curve dual (in B) to J_B

$$c_L^{(2)} = c_R^{(2)} = 3N^2 C \cdot C$$

- Subleading order: CS-coupling of 7-branes:

$$c_L^{(1)} - c_R^{(1)} = 6N c_1(B_2) \cdot C \,.$$

and level of R-symmetry $k_R = c_R/6$ from gauging of the $SO(4)_T$ isometry of the S^3 ($M = 1$)

$$k_R^{(1)} = \frac{1}{2} N c_1(B) \cdot C$$

- Central Charge:

$$c_L^{AdS} = 3N^2 C \cdot C + 9N c_1(B) \cdot C$$
$$c_R^{AdS} = 3N^2 C \cdot C + 3N c_1(B) \cdot C \,.$$

- Comparison to spectrum of $N = 4$ SYM on $\mathbb{R}^{1,1} \times C$ with duality twist

  $$c_L^{spec} = 3C \cdot C + 9c_1(B) \cdot C + 6\,, \qquad c_R^{spec} = 3C \cdot C + 3c_1(B) \cdot C + 6\,.$$

  Spectrum computation includes center of mass mode $(c_L, c_R) = (4, 6)$, which decouples in the IR:

  $$c_L^{AdS}|_{N=1} = 3C \cdot C + 9c_1(B) \cdot C + \ldots$$
  $$c_R^{AdS}|_{N=1} = 3C \cdot C + 3c_1(B) \cdot C\,.$$

  matches spectrum for $N = 1$ in first two leading orders. c_R exact result (see also match with self-dual string anomaly in 6d), but c_L gets corrections of $O(1)$.

Cross-checks:

M5-brane anomaly polynomial: $I_4 = \int_P I_8$ using approach in [Freed, Harvey, Minasian, Moore]. We find agreement with the above result.

Summary and Outlook

New chiral 2d SCFTs from dimensional reduction of $N = 4$ SYM with varying τ.

$G = U(1)$: complete understanding of spectrum, defect modes, central charges, using 6d point of view and geometry of singular fibers.

Physics workaround: Use holographic duality to compute central charges for general $U(N)$ gauge groups.

Nice upshot: New holographic framework of AdS/CFT within F-theory.

General G: Duality-twisted Hitchin system, however, how are duality defects characterized?

Math question:
characterize the duality-twisted (generalized) Hitchin moduli space including duality defects; possibly useful to utilizing T-duality/mirror symmetry induced by S-duality action on τ.